

# Examining Student Work for Mathematical Thinking (grades 3-8)

**Ann Gaffney**  
Londonderry Middle School and Rivier University  
Presented to: Sacred Heart, Hampton  
March 4, 2016  
gaffneyedcons@gmail.com

## Agenda



1. Introduction
2. A 5<sup>th</sup>/6<sup>th</sup> grade fractions writing prompt
3. Student-generated calculation methods
4. MARS formative assessment tasks (you choose your grade level and task)
5. Wrap-up

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given. 2

## Examining Student Work

**Why, when adding and subtracting decimals, must you line up the decimal points?** You may use pictures to help you explain, but you must also explain in words.

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given. 3

### Steven

- Procedural only
- Focused on the answer
- Example shows procedure

Why, when adding and subtracting decimals, must you line up the decimal points? You may use pictures to help you explain, but you must also explain in words.

You have to line up the decimal point because if you don't, you could mess up your problem. You could accidentally think that .46 is 46, and then you can't get your problem correct.

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given. 4

### Anna

- Conceptual
- Focused on place value
- Uses base ten blocks in picture but doesn't explain them

Why, when adding and subtracting decimals, must you line up the decimal points? You may use pictures to help you explain, but you must also explain in words.

You must line up the decimal points because tenths and hundredths don't have the same value, so when you add or subtract them, you don't know what to call the outcome. So, you can't add 1 tenth to 1 because 1 tenth is just a fraction of the 1 whole.

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given. 5

### Caitlin

- Conceptual
- Focused on place value
- Money example in the text matches her calculation example

Why, when adding and subtracting decimals, must you line up the decimal points? You may use pictures to help you explain, but you must also explain in words.

When adding and subtracting decimals, you must line up the decimals because of place value. You can't add 5 hundredths and 3 tenths, that would be like going to a bank and exchanging 9 pennies and a ten dollar bill for a hundred dollar bill.

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given. 6

### Student Thinking

- All 3 students could add decimals correctly.
- The differences were in their level of thinking about the problem conceptually.
- Student thinking tells us about student understandings and student misconceptions.

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given.

7

### Jennifer

$$a/b = a \div b$$

- $1 \div n$  is correct
- What happens when it becomes  $2 \div n$  ?

Think logically to answer each question below. Use manipulatives as cookies if you need to.

1. You have 1 cookie and are dividing it up among 3 people. How much of a cookie does each person get? Draw a picture to prove you are correct.  $\frac{1}{3}$
2. You have 1 cookie and are dividing it up among 4 people. How much of a cookie does each person get? Draw a picture to prove you are correct.  $\frac{1}{4}$
3. You have 1 cookie and are dividing it up among 5 people. How much of a cookie does each person get? Draw a picture to prove you are correct.  $\frac{1}{5}$
4. You have 1 cookie and are dividing it up among 26 people. How much of a cookie does each person get?  $\frac{1}{26}$
5. Now you have 2 cookies and are dividing them among 3 people. How many cookies does each person get? Draw a picture to prove you are correct.  $\frac{2}{3}$   $2 \div 3 = \frac{2}{3}$
6. You have 2 cookies and are dividing them among 4 people. How many cookies does each person get? Draw a picture to prove you are correct.  $\frac{2}{4} = \frac{1}{2}$

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given.

8

### Student Thinking

- Jennifer has a misconception. What is it?
- Jennifer is confused about the whole. Is one cookie the whole or is the group of cookies the whole?
- Student thinking tells us about student understandings and student misconceptions.

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given.

9

### A Fractions Writing Prompt

Explain why  $5 \times \frac{5}{6}$  equals  $4 \frac{1}{6}$ .  
 You may not use the algorithm to explain.  
 You must explain why the answer actually makes sense.

**In groups of 2-3, examine the index cards and discuss the questions on the back.**

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given.

10

### Student-Generated Calculation Methods

(Mostly student-generated anyway....)

- Posted around the room are student-generated calculation methods. **All of these methods DO work.**
- **Begin anywhere.** Look at each method and try to figure out what the student was thinking.
- **Can you figure out why each method works?**

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given.

11

### Why do these methods work?

© 2016 Gaffney Educational Consulting. Teachers may use and reproduce when there is no financial gain. Credit must be given.

12

A

$$\begin{array}{r}
 276 \\
 + 89 \\
 15 \\
 15 \\
 2 \\
 365
 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

13

B

$$\begin{array}{r}
 276 \\
 + 1189 \\
 365
 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

14

C

$$\begin{array}{r}
 276 \rightarrow 265 \\
 + 89 \rightarrow 100 \\
 365
 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

15

D

$$\begin{array}{r}
 29 \\
 ~~30~~^{10} 7 \\
 - 284 \\
 2723
 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

16

E

$$\begin{array}{r}
 3007 \\
 - 284 \\
 16 \\
 + 700 \\
 + 2000 \\
 + 7 \\
 2723
 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

17

F

$$\begin{array}{r}
 3007 \rightarrow 3023 \\
 - 284 \rightarrow 300 \\
 2723
 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

18

G

$$\begin{array}{r} 3 \ 10 \ 10 \ 7 \\ - \ 1 \ 21 \ 8 \ 4 \\ \hline 2 \ 7 \ 2 \ 3 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

19

H

$$\begin{aligned} &126 \times 48 \\ &= 252 \times 24 \\ &= 504 \times 12 \\ &= 1008 \times 6 \\ &= 6,048 \end{aligned}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

20

I

$$\begin{aligned} &13 \times 14 \\ &= (12 \times 12) + (12 \times 2) + 14 \\ &= 144 + 24 + 14 \\ &= 182 \end{aligned}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

21

J

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \\ + \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \hline 1 \ 4 \ 8 \ 2 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

22

K

$$\begin{aligned} &57 \times 26 = \\ &(50 + 7)(20 + 6) = \\ &1000 + 300 + 140 + 42 = \\ &1482 \end{aligned}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

23

L

$$\begin{aligned} &2/3 \div 4/5 = \\ &10/15 \div 12/15 = \\ &10/12 = \\ &5/6 \end{aligned}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

24

## Other methods you have seen....

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

25

M

$$\begin{array}{r}
 1 \quad 2 \quad 6 \\
 0 \quad 0 \quad 4 \quad 0 \quad 8 \quad 2 \quad 4 \quad 4 \\
 6 \quad 2 \quad 0 \quad 8 \quad 1 \quad 1 \quad 6 \quad 4 \quad 8 \quad 8 \\
 0 \quad 4 \quad 8
 \end{array}$$

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

26

## Why do we care?

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

27

## Why do we care?

- Diverse populations mean diverse algorithms.
- Student-generated calculation methods allow us to see their thinking.
- As teachers we must evaluate students' computation methods for their validity.
- Other algorithms may be equally as efficient as the traditional ones.

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

28

## Honor differences, develop shared understandings...

- Alternative and student-generated calculation methods are not necessarily wrong, and should be judged according to:
  - Applicability
  - Efficiency
  - Student understanding and facility with algorithm
- **Always teach the traditional algorithms to mastery while honoring alternative methods!**

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

29

## Examine a MARS Task

(MARS – Mathematics Assessment Resource Service)

These analyses of student work and resources for teachers available on the Inside Mathematics website:  
[www.insidemathematics.org](http://www.insidemathematics.org)

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

30

## Examine a MARS Task

1. Find a group of 3-4 from the same grade level.
2. Send a representative to choose a task.
3. Peruse the packet to see what is inside.
4. **Discuss the student work.**

**What do you notice about student understandings and misunderstandings?**

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

31

## The Take-Home Message

- We can “see” students’ thinking in their written work and calculation methods
- Examining student work for student thinking allows us to see:
  - student understandings
  - hidden misconceptions

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

32

## Resources

- The MARS Assessment Tasks used in this workshop are free to download from the Inside Mathematics website:  
**[www.insidemathematics.org](http://www.insidemathematics.org)**
- Today’s presentation handout:  
**[tinyurl.com/SacredHeartMath2016](http://tinyurl.com/SacredHeartMath2016)**
- Email me: Ann Gaffney at  
**[gaffneyedcons@gmail.com](mailto:gaffneyedcons@gmail.com)**

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

33



**Ann Gaffney**

28 Tokanel Dr.  
Londonderry, NH  
03053

(919) MATH – GEC

**[gaffneyedcons@gmail.com](mailto:gaffneyedcons@gmail.com)**

© 2016 Gaffney Educational Consulting.  
Teachers may use and reproduce when there is no financial gain. Credit must be given.

34