CHANGE IN EXPERIENCED TEACHERS' PEDAGOGICAL BELIEFS THROUGH LEARNING ELEMENTARY MATHEMATICS CONTENT

BY

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Abstract

This qualitative case study examined the connection between experienced teachers' pedagogical beliefs and their learning of mathematics content. The beliefs of eight experienced elementary (K-8) mathematics teachers were examined before, during, and after the teachers participated in a professional development training exclusively teaching elementary mathematics content. Teachers' beliefs about quality mathematics lessons were solicited through lesson plans, journals, and interviews. Research questions discussed are: (1) What do experienced K-8 teachers believe constitutes a "quality mathematics lesson?" (2) How does the experience of learning mathematics content through inquiry change teachers' beliefs about what constitutes a "quality mathematics lesson?" This study found that teachers changed their beliefs about quality lessons with regard to mathematics content, to pedagogical strategies, and to students as learners through their experience learning mathematics. Teacher beliefs became more focused on mathematical reasoning, more focused on inquiry, and more student-centered. These new beliefs better align with definitions of quality instruction from the literature. Teachers incorporated their beliefs about mathematics, pedagogical strategies, and students as learners into a vision of quality mathematics lessons and the teacher's role in creating those lessons. Teachers' vision of their role changed from that of provider of knowledge to a guide of student discovery of mathematical understandings. The data indicated that these changes in beliefs, including changes in beliefs about pedagogy, were driven by the act of learning mathematics content via methods of inquiry. The results of this study have implications for understanding current and future research on teacher beliefs, for in-service professional development training in mathematics teaching, and for improving teacher effectiveness and student achievement in mathematics.

Keywords: teacher learning; pedagogical beliefs; mathematics content knowledge; mathematics teachers; inquiry; constructivist teaching methods

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Chapter I: Introduction

Teacher effectiveness is an important component of student achievement. The teacher to which a child is assigned is the largest predictor of that student's future success in mathematics (Sanders & Horn, 1998). Further, the impact of an ineffective mathematics teacher[1] can be seen in student achievement data three years after the low-quality instruction occurred (Sanders & Horn, 1998). Given these facts, it becomes clear that one important way to improve student achievement in mathematics is to improve teacher effectiveness.

One characteristic that influences a teacher's ability to provide quality instruction is the teacher's level of content knowledge (Hill, Rowan, & Ball, 2005). Teachers in the United States often lack the knowledge they need in order to teach mathematics effectively (Ball, 1990; Ma, 1999) and this lack of teacher content knowledge is one factor in the failure to see expected achievement gains despite mathematics education reform efforts (Ross, McDougall, & Hogaboam-Gray, 2002). For example, Hill, Rowan, and Ball (2005) found that the achievement of students taught by highly mathematically knowledgeable teachers was significantly higher than those who were taught by teachers with average mathematical knowledge for teaching (MKT).[2] The difference in MKT had as strong an effect on student achievement as the

[1] Sanders & Horn (1998) considered an "ineffective teacher" of mathematics to be a teacher in the bottom 20% as measured by cumulative average student gain on the tests that are part of the Tennessee Value-Added Assessment System.

[2] Hill, Rowan, and Ball (2005) separated teachers into quartiles according to their scores on their measures of mathematical knowledge for teaching (MKT), a special type of teacher knowledge including knowledge of both mathematics content and pedagogical knowledge specific to the teaching of mathematics. Teachers that I call "highly mathematically knowledgeable" are those who scored in the top quartile of teachers.

socioeconomic status of the student or the equivalent of up to three weeks of instructional time.

Despite its importance, increasing the content knowledge of teachers does not single-handedly improve instruction. Teachers' beliefs and goals impact their instruction (Bray, 2011; Liljedahl, 2010; Thompson, 1984). In addition, the positive effects of content knowledge can be diminished by other factors, including teacher beliefs about mathematics instruction (Hill et al., 2008; Provost 2013; Raymond, 1997). In order to understand these interactions it is important to understand what teachers believe about lesson quality.

Although there is a significant body of literature about the characteristics of quality mathematics lessons (e.g. Ball, Thames, & Phelps, 2008; Hill et al., 2008), there is little in the research literature about what *teachers* believe makes a quality lesson (Wilson, Cooney, & Stinson, 2005). Teachers create lessons and choose the instructional strategies used to teach their lessons. In the 2012 National Survey of Science and Mathematics Education (NSSME), 44% of elementary teachers and 70% of middle school teachers[3] surveyed said they felt strong control[4] over the teaching techniques selected for their mathematics classes (Banilower et al., 2013). What teachers consider "quality" is one important factor impacting the instructional strategies a teacher chooses to use for mathematics instruction.

In their analysis of the NSSME, Banilower et al. (2013) did examine teachers' responses to various instructional belief statements, but they may not have examined

[3] Elementary teachers in the NSSME study taught grades 1-5 or in a self-contained 6th grade classroom. Middle school teachers taught non-self-contained 6th grade or grades 7-8.
[4] The NSSME report did not define "control."

teachers' beliefs about quality lessons with enough depth to understand how teachers' beliefs may be expressed in their mathematics lessons or how those beliefs may change over time. Wilson, Cooney, and Stinson (2005) studied experienced teachers' perceptions of good mathematics teaching and how teachers believe good *teaching* develops, but they did not study how teachers' *beliefs* develop and change. This study adds to the literature base by examining two qualitative research questions related to teachers' beliefs about lesson quality: (1) What do experienced K-8 teachers believe constitutes a "quality mathematics lesson?" (2) How does the experience of learning mathematics content through inquiry[5] change teachers' belief statements about what constitutes a "quality mathematics lesson?"[6]

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[5] Inquiry, in this study, describes the investigation of mathematical ideas (Schifter, 1996a).

[6] Initially, the study had three research questions. These questions changed as the study progressed. The research questions listed here are the revised research questions.

Chapter II: Review of the Literature

This study examined the interaction between teacher knowledge and teacher beliefs about quality mathematics lessons as teachers learned elementary mathematics content. Understanding this interaction requires background in four key areas: teacher knowledge, teacher beliefs, quality mathematics instruction, and teacher learning. This chapter presents an overview of the literature in each area, relevant theoretical frameworks used in this study, and empirical studies that specifically inform the discussion of this study's results and accompanying interpretation.

Teacher Knowledge

Two different frameworks of teacher knowledge were used during this study. The first framework, the content–pedagogy framework (C/P), was used in designing the study and the beginning stages of data collection. The second framework, Darling-Hammond and Bransford's (2005)[7] *framework for understanding teaching and learning* (UTL), was used in the analysis of the data. The UTL framework was added once it became clear that teachers' knowledge and beliefs about students as learners were an important component of the data.

A framework for teacher knowledge used in the study design. The content–pedagogy framework (see Figure 1) was used during the design of the study and the initial stages of data collection and analysis. In the C/P framework, teacher knowledge includes knowledge of both content and pedagogy. Effective teaching requires

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[7] The framework for understanding teaching and learning is presented in the introduction (Bransford, Darling-Hammond, & LePage, 2005) of *Preparing Teachers for a Changing World: What Teachers Should Know and are Able to Do* (Darling-Hammond & Bransford, 2005). The framework was used as the framework for the entire text and was elaborated upon within each chapter of the book. Because of this, I have cited the framework in the text as part of the book as a whole rather than just the introductory chapter.

knowledge of both content and pedagogy; lack of pedagogical knowledge makes it difficult for mathematics teachers to impart knowledge to students (e.g. Even & Tirosh, 1995), while lack of adequate content knowledge may result in teaching that contains mathematical flaws (e.g. Ma, 1999; Putnam, Heaton, Prawat, & Remillard, 1992).

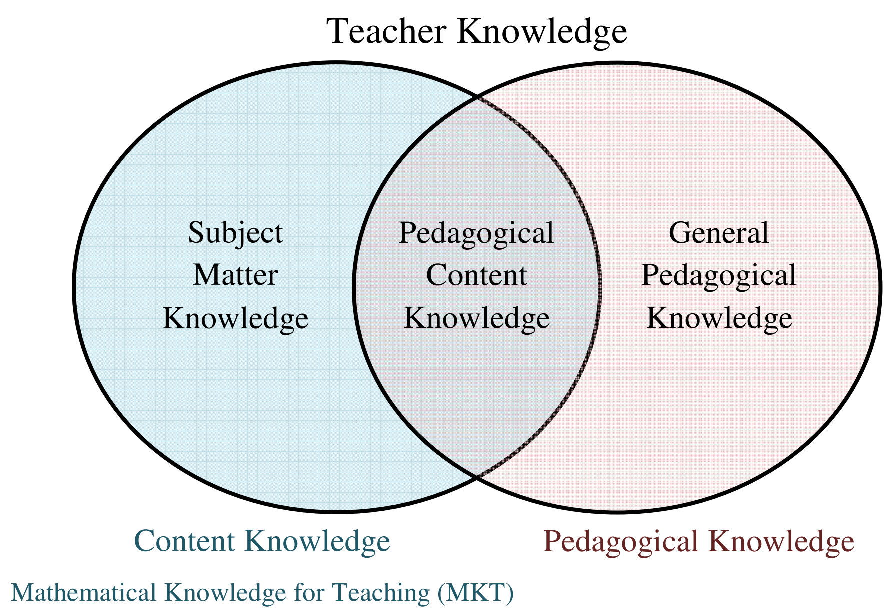

Figure 1. The Content–Pedagogy Framework. This framework organizes types of teacher knowledge. The term content knowledge is often used to describe the entire left circle made up of both subject matter knowledge and pedagogical content knowledge. These also make up mathematical knowledge for teaching (MKT). The term pedagogical knowledge is often used to describe the right circle made up of both pedagogical content knowledge and general pedagogical knowledge.

Knowledge of content and knowledge of pedagogy are not distinct. These two categories of teacher knowledge overlap in an area called pedagogical content knowledge (Shulman, 1986, 1987). Pedagogical content knowledge represents the "blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8).

Due to this overlap, it is useful to be able to discuss areas of content knowledge and areas of pedagogical knowledge that do not include pedagogical content knowledge. *Mathematical knowledge for teaching* (MKT; Ball et al., 2008) provides one way to categorize the content knowledge circle of the content–pedagogy framework. MKT delineates two main areas of content knowledge: subject matter knowledge and pedagogical content knowledge.[8] Subject matter knowledge is content knowledge that does not include pedagogical content knowledge. Subject matter knowledge consists of the procedural and conceptual mathematics that teachers must know. For example, teachers must know how to correctly add and subtract fractions and know why one needs common denominators for addition and subtraction of fractions.

Pedagogical content knowledge refers to the knowledge about the ways that mathematics is taught and learned. Teachers use pedagogical content knowledge when they choose, for example, whether to use a trading model[9] or a bundling model[10] for teaching place value. The term content knowledge is often used to describe the body of

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8 MKT further subdivides subject matter knowledge and pedagogical content knowledge. These divisions were not used for the analysis of the data in this study and so are left out of the discussion here.

9 In a trading model, students collect ten items with a value of one (e.g. a penny or a unit cube) and physically trade them in for one item with a value of ten (e.g. a dime or a long block with the same volume as ten unit cubes).

10 In a bundling model, students collect ten items with a value of one (e.g. straws or attachable cubes) and group them together to create a bundle with a value of ten (e.g. ten straws with a rubber band wrapped around them or ten cubes attached together).

knowledge made up of both subject matter knowledge and pedagogical content knowledge.

General pedagogical knowledge is the part of the knowledge of pedagogy circle in the C/P framework that does not include pedagogical content knowledge. General pedagogical knowledge is not specific to mathematics and is not a part of MKT, but is a part of the C/P framework used in this study. It includes knowledge of general teaching techniques, such as classroom management strategies or learning styles, that are used in the teaching of all content areas. The body of knowledge made up of both pedagogical content knowledge and general pedagogical knowledge is often referred to simply as pedagogical knowledge.

An expanded framework for teacher knowledge. In analyzing and interpreting the data collected in this study, I found that there were limitations to the content–pedagogy framework. The content–pedagogy framework was not inaccurate, rather it was incomplete. I needed a framework that incorporated knowledge of students and the ways in which students learn in general, not just in mathematical contexts. Darling-Hammond and Bransford's (2005) framework for understanding teaching and learning expands the content–pedagogy framework to incorporate students and organizes the different kinds of "knowledge, skills, and dispositions that are important for any teacher to acquire" (Bransford, Darling-Hammond & LePage, 2005, p. 10).

The framework for understanding teaching and learning (see Figure 2) is made up of three intersecting circles representing knowledge of subject matter and curriculum goals, knowledge of teaching, and knowledge of learners and their development in social contexts. For consistency, when describing these types of knowledge in the dissertation, I use shortened titles that better match the terms used by other researchers and the study participants. The shortened titles for these types of knowledge are knowledge of mathematics content, of pedagogy, and of students, respectively. These three areas overlap and influence one another. For example, what one knows about students as learners one uses to teach students subject matter. This overlap is evident in UTL as the center part of the framework where all three circles intersect. This area, what Bransford et al. (2005) call *a vision of professional practice*, represents how teachers use all their types of knowledge when teaching students mathematics. Each of these circles also includes the social context surrounding the subject matter, teaching, and learners. These three circles define the learning environment.

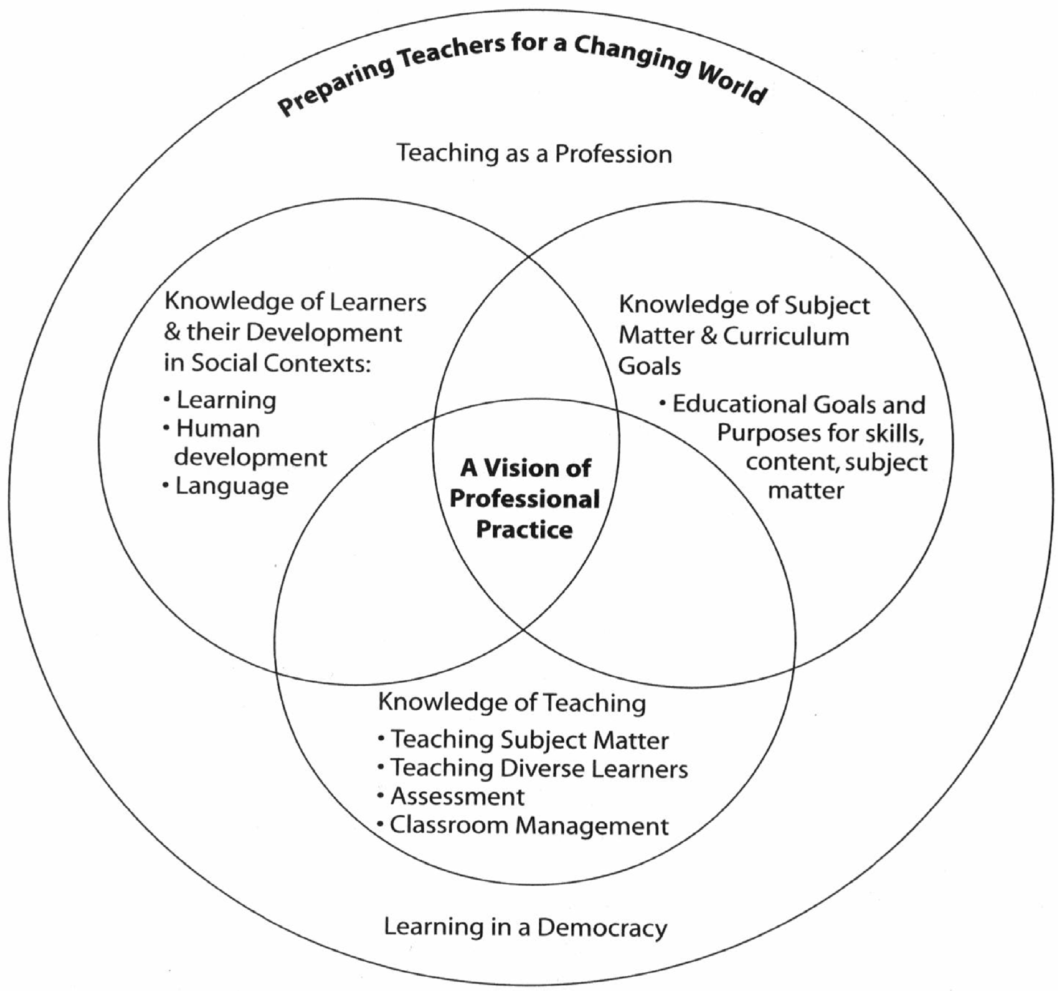

Figure 2. A Framework for Understanding Teaching and Learning. From *Preparing Teachers for a Changing World* (p. 11), Edited by L. Darling-Hammond and J. Bransford, 2005, San Francisco: Jossey-Bass. Copyright 2005 by John Wiley & Sons, Inc. Reprinted with permission.

The content–pedagogy framework used in the beginning of the study is a subset of the framework for understanding teaching and learning. The content knowledge circle in C/P corresponds to the knowledge of subject matter and curriculum goals in UTL; the pedagogical knowledge circle in C/P corresponds to the knowledge of teaching circle in UTL. In addition, the UTL framework adds a third circle, the knowledge of learners and their development in social contexts. This third circle was necessary to represent teachers' belief statements and was the impetus behind the change in frameworks during data analysis. The UTL framework is especially useful for this study because the three areas of teacher beliefs and five areas of mathematical quality of instruction, discussed below, also map onto UTL (see Appendix A).

Components of teacher knowledge. Teacher knowledge is composed of three intersecting types of knowledge: knowledge of mathematics content, of pedagogy, and of students. Although these types of knowledge intersect, they can also be distinguished from each other. Each of these types of knowledge is discussed below.

Knowledge of mathematics content.

Assuming that the content of first-grade mathematics is something any adult understands is to doom school mathematics to the dull, rule-based curriculum that is so widely criticized.

– Deborah Loewenberg Ball (1990, p. 462).

The knowledge of mathematics needed for teaching is different than the knowledge of mathematics needed by the layperson (Ball, 1990). This knowledge involves more than knowledge of calculations and procedures. Knowing mathematics for teaching at the elementary level requires an understanding of elementary mathematics that is typically not taught as part of a mathematics major at the university level (Ball, 1990). This study examined experienced, practicing teachers' understanding[11] of elementary mathematics and examine changes in beliefs about lesson quality that occurred during that process.

The National Research Council (Kilpatrick, Swafford, & Findell, 2001), in its widely referenced review of the research literature, *Adding It Up: Helping Children Learn Mathematics*, defines mathematical proficiency as comprised of five interdependent strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The first two, conceptual understanding and procedural fluency, are often thought of as the subject matter or content of mathematics and are discussed in more detail below. Strategic competence and adaptive reasoning refer to practices or skills used when doing mathematics. Productive disposition refers to one's attitudes and beliefs about mathematics and the learning of mathematics (e.g. one's belief that they can do mathematics). The Common Core State Standards (CCSS) for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) groups strategic competence, adaptive reasoning, and productive disposition into the Standards for Mathematical Practice. This grouping follows the tradition of thinking of mathematics as comprised of two areas: content, and practices or skills. Although these practices are also mathematical content, the term *mathematical practices* is often used when referring to this type of knowledge. The word *content* is used to mean procedural and conceptual knowledge.

Procedural and conceptual knowledge. There are two different types of mathematical content knowledge (Hiebert, et al., 1997; Nesher, 1986; Skemp,

[11] Understanding, in this study, means knowing not just concepts or ideas, but the ways in which those concepts or ideas are connected to other concepts and ideas. This definition stems from Hiebert and Carpenter's (1992) definition of understanding.

1976/2006; Shulman, 1986). These types of content knowledge have different names depending on the researcher. Essentially they include knowledge of the rules and procedures, and knowledge of the "underlying structure of mathematics" (Eisenhart et al., 1993, p. 9). The knowledge of calculation algorithms and when to use them is an example of the first type of content knowledge, the knowledge of the rules and procedures. This is what the National Research Council (Kilpatrick, Swafford, & Findell, 2001) means by procedural fluency. In the context of this study, knowledge of rules and procedures is called procedural knowledge.

Conceptual knowledge is an example of the second type, knowledge of the underlying structure of mathematics. Conceptual knowledge goes further than procedural knowledge. It involves a broadening of the ideas evident within an algorithm or within the field of mathematics to create a foundation for future content knowledge. Hiebert and Carpenter (1992) define understanding according to the number and strength of connections between a mathematical idea or procedure and other mathematical ideas or procedures and assert that "many of those who study mathematics learning agree that understanding involves recognizing relationships between pieces of information" (p. 67). Conceptual knowledge is sometimes referred to as conceptual understanding because knowledge of the underlying structure of mathematics involves knowledge of the connections between ideas.

It is important to distinguish between the use of the term understanding as defined above and common usage of the term understanding. Skemp (1976/2006) distinguishes between these two usages of understanding by calling them relational understanding and instrumental understanding. Relational understanding is understanding that involves “knowing both what to do and why” (Skemp, 1976/2006, p. 89) and corresponds to Hiebert and Carpenter’s (1992) connections definition above. Instrumental understanding is knowing what to do and when. This, according to Skemp (1976/2006), is what many students and their teachers mean when they say they “understand” something. In this dissertation, the word understanding refers to relational or conceptual understanding, not to instrumental understanding.

Teaching requires knowledge of both procedural and conceptual types of content knowledge (e.g. Hiebert et al, 1997; Shulman, 1986). The professional development used in this study focused on the conceptual understanding of mathematical content. Procedures were not explicitly taught. When procedural knowledge was discussed in the professional development, it was done so in concert with conceptual knowledge about the mathematical ideas behind the algorithms. Hence, the professional development focused on building connections between ideas and other ideas or procedures; it focused on conceptual understanding.

Deep vs. surface learning. Related to procedural and conceptual knowledge are the ideas of deep and surface knowledge. The terms deep and surface were first applied to learning by Marton and Säljö (1976). In their study, Marton and Säljö found that college students reading and studying a passage used one of two approaches to learning the content of the passage. Some students attempted to memorize information that they thought they would be questioned on later. This approach was called surface learning. Other students actively searched for meaning. This approach was called deep learning. Rote memorization of procedures can be learned using surface learning; forming connections among ideas requires deep learning (Davis & Renert, 2014). This means that procedural knowledge can be gained through surface learning, but conceptual knowledge must be gained through deep learning.

In mathematics education, the term “deep understanding” is used often (e.g. Ball, Hill, & Bass, 2005, National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; National Mathematics Advisory Panel, 2008) and is rarely defined. Bereiter (2006) writes:

> Everyone is in favor of depth. We use the term with confidence, even though we cannot define it, and evaluating it is highly subjective. We speak of depth of understanding and depth of feeling. A book or an art work may be deep, and so may our appreciation of it. In-depth analyses are always on offer. There can be depth of learning in any content area, any complex skill. There is depth in the treatment of concepts, issues, problems, and interpretations. In short, virtually all the more elevated educational objectives can be cast in terms of depth. (p. 11)

Garner (2007) defines deep understanding of mathematics as “not algorithmic prowess, but an ability to communicate mathematically, represent mathematics in different forms, and engage in problem-solving and reasoning mathematically” (p. 173). Notice the connection between Garner’s definition of deep understanding as including different forms of mathematical representations, and Hiebert and Carpenter’s (1992) definition of understanding as “recognizing relationships between pieces of information” (p. 67). The term deep understanding is used in mathematics education to mean using and recognizing relationships and connections between mathematical ideas or concepts.

Bereiter (2006) lists one of depth’s opposites as superficiality. In this dissertation, deep is the opposite of superficial. One’s understanding is deep if it employs more connections and hence is not superficial. One's beliefs are deep if they are held centrally and are not superficial. Although it may be argued that the term deep should be avoided due to its difficulty to define, the term is embedded in the field of mathematics education. Additionally, since teachers in this study use the term "deep" or "deepening" to describe their understandings and their beliefs, its use is unavoidable.

Knowledge of pedagogy. Knowledge of pedagogy includes knowledge of pedagogical strategies specific to the teaching of mathematics (pedagogical content knowledge) as well as pedagogical strategies used across content areas. The teachers in this study, all experienced teachers, entered the study with a repertoire of general and math-specific teaching strategies. Because this study focused on the teaching and learning of mathematics, general pedagogical strategies appeared only in their use in the teaching of mathematics. Hence, this discussion focuses on knowledge of pedagogy (both general and content-specific) as it is expressed in the context of mathematics teaching.

Traditional and constructivist philosophies of mathematics teaching and learning. Teaching and learning are related (Koehler & Grouws, 1992). The field of mathematics education in the United States can be characterized as having two main pedagogical philosophies for the learning and teaching of mathematics (Davis, Maher, & Noddings, 1990; Gage, 2009; Mewborn & Cross, 2007). These two schools of thought have been given many names; this dissertation calls them traditional and constructivist.

Traditional philosophies of teaching and learning stem from positivist traditions of knowledge where knowledge is gained about an absolute or objective truth. In this philosophy, mathematics represents a body of knowledge that can be transferred from the teacher to the student, who is a passive recipient of knowledge (Marlowe & Page, 2005). In a typical traditional classroom, "the teacher faces her students, who watch and listen as she broadcasts the day's algorithmic routine, posing questions to see if they are following her explanations, and repeating them if they are not. Then, having assigned a series of repetitive exercises to fix the routine in memory, she checks answers to establish that the point of the lesson has been understood (that is, that the routine has been accurately memorized) before moving on to the next topic" (Schifter, 1996b, p. 3).

Constructivist philosophies, in contrast to traditional philosophies, assert that knowledge is not merely transferred, but constructed. In their introduction to one of the National Council of Teachers of Mathematics' (NCTM) early books on constructivism, the editors write:

> As readers will discover, even those who label themselves constructivists have differences of opinion on theoretical issues and express preferences for some strategies [for the teaching of mathematics] over others. But a common thread runs through all chapters, namely, the emphasis on mathematical activity in a mathematical community. It is assumed that learners need to construct their own knowledge—individually and collectively. Each learner has a toolkit of conceptions and skills with which he or she must construct knowledge to solve problems presented by the environment. The role of the community—other learners and teacher—is to provide the setting, pose the challenges, and offer support that will encourage mathematical construction. (Davis et al., 1990, p. 3)

This stance by NCTM on constructivism, the consistency of NCTM's standards documents with constructivism (Marlowe & Page, 2005), and the changes the organization advocated through various reform efforts have caused some to label constructivist philosophies of the teaching and learning of mathematics as "reform-oriented" (e.g. Ross et al., 2002; Mewborn & Cross, 2007). Constructivism is not new, nor is it limited to the learning of mathematics (Davis et al., 1990; Fosnot, 1996; von Glasersfeld, 1990). There are many types of constructivists (e.g. radical constructivists, social constructivists, psychological constructivists) (Phillips & Soltis, 2009), but this study does not distinguish between them. This study uses the ideas common to these types of constructivism to contrast constructivist and traditional views of teaching and learning.

Constructivism is a way of thinking about learning. Goldin (1990) cautions against treating constructivism as a theory of teaching: "It is important to recognize that one does not need to accept radical constructivist epistemology in order to adopt a model [framework] of learning as a constructive process, or to advocate increased classroom emphasis on guided discovery in mathematics" (Goldin, 1990, p. 40). Fosnot (1996) asserts that "although constructivism is not a theory of teaching, it suggests taking a radically different approach to instruction than is used on most schools" (p. ix). In this study, the term constructivist describes beliefs that quality learning stems from the types of activities that are consistent with the idea that learning as a constructive process.

Definitions of traditional and constructivist teachers. In this study, the following descriptions define the characteristics of teachers who are to be considered traditional and constructivist. These definitions draw on the work of Beswick (2005; 2007) and Raymond (1997), both of whom have done empirical research using similar descriptions. These descriptions represent ends of a spectrum. Teachers may fall anywhere along the spectrum, and their placement on that spectrum may move over time or in different contexts (Beswick, 2005; Raymond, 1997).

Traditional teachers are teachers who believe their role is to teach students the ways to do mathematics. Traditional teachers teach primarily from a textbook, instruct students in procedures and methods to answer questions, use a predominantly lecture format, and emphasize memorizing and practicing procedures. In a traditional classroom, teachers do most of the talking.

Constructivist teachers are teachers who believe their role is to guide students in constructing conceptual understandings about mathematical ideas. Constructivist teachers rarely teach from a textbook and primarily use problem solving tasks where students discover mathematical ideas while working in groups. Student groups develop their own procedures and use those procedures for future work. In a constructivist classroom, students do most of the talking.

Inquiry. Constructivist philosophies of teaching and learning require students to build mathematical knowledge. With the introduction of constructivist ideas into mathematics instruction "the mathematics classroom was to become a community of inquiry, a problem-posing and problem-solving environment in which developing an approach to thinking about mathematical issues would be valued more highly than memorizing algorithms and using them to get the right answers" (Schifter, 1996a, p. 78). In this study the term "inquiry" is used to describe the investigation of mathematical ideas. Methods of inquiry are used to build understanding of mathematical ideas. Batista (1990) describes: "In an appropriate classroom culture of inquiry, students, like scientists, construct, revise, and refine theories to solve and make sense of perceived problems" (p. 447). Methods of inquiry are consistent with constructivist ideas about learning (Battista, 1990; Schifter, 1996a). The professional development in this study used methods of inquiry to explore mathematics.

Experts and novices. Experts is a term used to describe individuals who know a topic very well and are skilled in their area of expertise. Experts are not only able to efficiently perform tasks, but can innovate and apply their knowledge to novel situations (Schwartz, Bransford, & Sears, 2005). In their exhaustive review of the literature on how learning occurs, Bransford, Brown, and Cocking state that "expert teachers know the kinds of difficulties that students are likely to face; they know how to tap into students' existing knowledge in order to make new information meaningful; and they know how to assess their students' progress" (2000, p.45). In reviewing the literature on the differences between expert and novice teachers, Brown & Boyko summarized: "Expert teachers displayed more pedagogical knowledge, content knowledge, and pedagogical content knowledge than did novices. Further, their conceptual systems, or cognitive schema, for organizing and storing this knowledge are more elaborate, interconnected, and accessible than the novices' schemata" (1992, p. 213). In other words, the learning of experts was deeper than that of novices.

Leinhardt & Smith (1985) distinguished between expertise in lesson structure, a form of pedagogy, and expertise in subject matter or content. In the analysis of the data, it became clear that the analysis needed to distinguish between teachers' varying levels of content expertise and varying levels of pedagogical expertise. Some researchers argue that teachers move through stages as they develop pedagogical expertise (Berliner, 1991), while others argue that a stage model implies mastery of some skills as a necessary precursor to mastery of others and that this implication is false (Grossman, 1992); both agree that teachers do develop pedagogical expertise over the course of their careers. Berliner (2004), in his examination of the literature on the development of teaching expertise wrote that "a reasonable answer to the question of how long it takes to acquire high levels of skill as a teacher might be 5 to 7 years if one works hard at it. Competence as a teacher might come about 2 years earlier" (p. 201). The teachers in this study, all of whom had at least three years of teaching experience, had developed a level of pedagogical expertise in *teaching*, distinct from their level of expertise in *mathematics*.

Knowledge of students. Students come to the mathematics classroom with a diversity of backgrounds, abilities, prior knowledge, learning preferences, and confidence levels. Teachers use their knowledge of students, both knowledge of particular students and knowledge of students in general, every day in their classrooms. Examples of knowledge of students include knowledge of human development, knowledge of the diverse cultural and linguistic backgrounds of students and the ways in which those backgrounds impact interactions with others, and knowledge of student safety and classroom climate. Teachers use knowledge of students when they judge students' comprehension of a lesson or tailor a lesson to include student interests.

Knowledge of pedagogy is closely related to knowledge of students' learning (Koehler & Grouws, 1992). Teachers' knowledge about the ways in which students learn influences the ways in which teachers work with students.[12] Expert teachers, when removed from their own classrooms and schools where they have knowledge of

[12] This is also evident in the discussion above about pedagogical philosophies of teaching and learning.

students and routines, judge themselves to be less effective (Berliner, 2004). Their expertise in teaching depends, in part, on knowledge of students.

Initially, this study did not include an examination of this domain of teacher knowledge, other than the areas of knowledge about students that overlapped with knowledge about mathematics content or mathematics teaching strategies. During the study, teachers discussed issues related to students that were not unique to mathematics (e.g. issues of classroom climate, learning styles, homework). As a result, this domain of knowledge about students was added to the study.

Teacher Beliefs

The literature on beliefs prompted Skott (2001) to describe beliefs as what they are not. Beliefs, he said, "may be described in negative terms as not requiring standardized canons of evidence, not requiring common agreement, and not requiring internal consistency" (Skott, 2001, p. 6). What beliefs *are* is much more difficult. Green (1971) wrote “a complete analysis of any really basic concept like ‘knowledge’ or ‘belief’ is almost never attained” (p. 15). Pajares (1992) discussed at length the confusion in the literature between beliefs and knowledge, concluding that “the chosen and perhaps artificial distinction between belief and knowledge is common to most definitions: Belief is based on evaluation and judgment; knowledge is based on objective fact” (p. 313).

I began this study with Pajares’ distinction between beliefs and knowledge and defined beliefs as the views or ideas about a given topic based on evaluation and judgment. As the study progressed, I began to see the trouble with separating knowledge and belief as expressed by Green (1971). For example, one study participant, Bethan, believed that a multimodal approach allowed her students to access learning. If this matches the research literature, is it knowledge, or is it belief? Does it matter whether or not Bethan knows the research literature on the issue? If she does not, she is basing her statement on evaluation and judgment; if she does, what level of certainty is necessary before she can call her belief "knowledge?" It is issues like these that illustrate the difficulty in defining beliefs for the purposes of this research study.

One way that researchers have made sense of this issue is to consider knowledge one aspect of belief (e.g. Beswick, 2007; Leatham, 2006). Leatham (2006) explains "Of all the things we believe, there are some things that we 'just believe' and other things that we 'more than believe – we know.' Those things we 'more than believe' we refer to as knowledge and those things we 'just believe' we refer to as beliefs. Thus beliefs and knowledge can profitably be viewed as complementary subsets of the set of things we believe" (p. 92). I have adopted the definition of beliefs used by Beswick (2007) who considers beliefs to be "anything the individual regards as true" (p. 96). This definition allows the distinction between beliefs and knowledge to be made by the believer and her level of certainty about the belief, rather than by society's recognition of the belief as "objective fact."

Pajares (1992) suggests that the idea of teacher beliefs as one entity is not clear enough for research purposes and suggests limiting studies of teacher beliefs to teachers' "educational beliefs about" (p. 316) a particular topic. The National Council of Teachers of Mathematics (NCTM) uses three main categories (see Figure 3), grounded in empirical research (e.g. Beswick, 2007; Raymond, 1997), when discussing the educational beliefs of mathematics teachers: beliefs about mathematics, beliefs about students as learners, and beliefs about pedagogical strategies (NCTM, 2000).

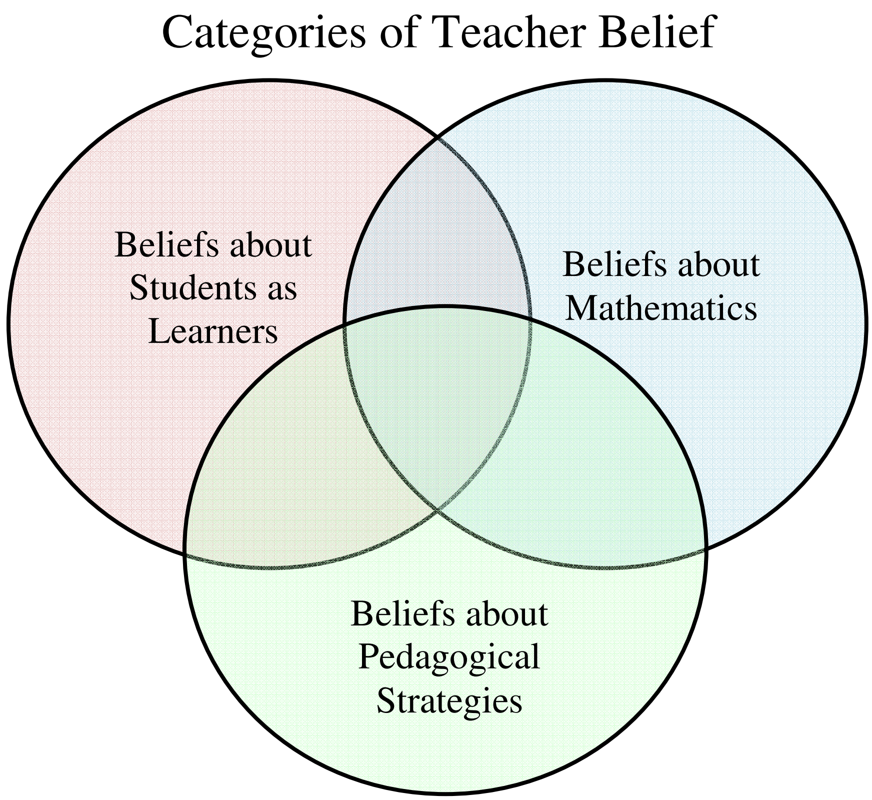

Figure 3. Categories of teacher's educational beliefs.

These categories may overlap and influence one another, but are useful ways to separate and examine different facets of teacher beliefs. Note that these three categories of beliefs correspond directly to the three areas of teacher knowledge described above.

Beswick (2005) presents a relationship between beliefs across these three areas (see Table 1). She examined Ernest's (1989, as cited in Beswick, 2005) delineation of three types of beliefs about the nature of mathematics and beliefs about mathematics learning alongside Van Zoest, Jones, and Thornton's (1994) three types of beliefs about mathematics teaching. Beswick matched these three areas of beliefs so that "beliefs on the same row are regarded as theoretically consistent with one another, and those in the same column have been regarded by some researchers as a continuum (Anderson & Piazza, 1996; Perry et al., 1999; Van Zoest et al., 1994)" (2005, p. 40).

Table 1

Relationships Between Beliefs

Beliefs about the nature of mathematics (Ernest, 1989b)	Beliefs about mathematics teaching (Van Zoest et al., 1994)	Beliefs about mathematics learning (Ernest, 1989b)
Instrumentalist	Content-focused with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content-focused with an emphasis on understanding	Active construction of understanding
Problem-solving	Learner-focused	Autonomous exploration of own interests

Note. From "The beliefs/practice connection in broadly defined contexts," by K. Beswick, 2005, *Mathematics Education Research Journal, 17*(2), p. 40. Copyright 2005 by Springer. Reprinted with permission.

In this context, the term instrumentalist describes those who view mathematics as a collection of rules and facts; platonists view mathematics as a body of knowledge to be discovered; while those with a problem-solving view of mathematics view it something that humans create (Ernest, 1988). Beliefs in the second and third rows of Table 1 would both be described as constructivist because they involve beliefs that knowledge or understanding is constructed by the individual. This study primarily focuses on teachers' beliefs about mathematics teaching. Beswick's alignment is useful in this study for examining teachers' pedagogical beliefs in the context of their interaction with other types of beliefs, particularly those surrounding the nature of mathematics learning as well as teaching.

Many researchers (e.g. Cooney, 1985; Fang, 1996; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1984) have discussed the consistency or inconsistency of teacher beliefs and their classroom practices. Thompson (1984) studied junior high math teachers to determine their beliefs about mathematics and mathematics instruction and if those beliefs influenced their instruction. Thompson did not describe common beliefs, but did find that teachers held beliefs about instruction that were both general and specific to mathematics and that the general beliefs about instruction may "take precedence over other views and beliefs specific to the teaching of mathematics" (p. 125).

Raymond (1997) conducted a school-year length study of six elementary teacher's beliefs about content and pedagogy, and presented the results of one case study in this article. The teacher's beliefs about mathematics content and beliefs about pedagogy were not consistent. The teacher's beliefs about content were more traditional than her beliefs about pedagogy, but her instruction was more closely aligned with her expressed beliefs about content than with her expressed beliefs about the ways in which the content should be taught. "Results of this study suggest that deeply held, traditional beliefs about the nature of mathematics have the potential to perpetuate mathematics teaching that is more traditional, even when teachers hold nontraditional beliefs about mathematics pedagogy" (Raymond, 1997, p. 574).

These inconsistencies in teachers' beliefs imply complexities of implementing beliefs. Beswick (2005) and Leatham (2006) both argue that it is important to consider contexts when examining beliefs and that the inconsistencies between teachers' beliefs and practice noted in other studies can be accounted for by the differing contexts in which those beliefs were examined. Beswick's and Leatham's works both draw on the work of Green (1971) who asserts that some beliefs are held more centrally than others and that when beliefs are in conflict, those beliefs more centrally held are expressed.

Beliefs can change. Beliefs also differ in their resistance to change (Chi, 2005; Savion, 2009). The nature of the change process is debated in the literature. Some researchers argue that students' performance and reactions to instruction impact teachers' beliefs (Guskey, 1986), while others assert that beliefs impact instruction (Liljedahl, 2010; Thompson, 1984). Ernest (1989) presents a model of mathematics teachers' knowledge, beliefs, and attitudes that consists of a cycle of planning, teaching, reflecting, and planning. The cyclical nature of the model suggests that beliefs may both impact instruction and be impacted by instruction.

Teacher beliefs about mathematics instruction may be fairly stable absent some novel factor driving change (Stipek, et al., 2001; Wood, Cobb, & Yackel, 1991). Stipek et al. (2001) studied twenty-one fourth through sixth grade teachers' beliefs and practices related to mathematics instruction. They found that teachers' beliefs about instruction were "fairly consistent over the course of a school year" (p. 222). Wood, Cobb, and Yackel (1991) studied one experienced second grade teacher as she "reflected on and resolved the conflicts created between her established ways of practice and the emphasis of the project on children's construction of mathematical meanings" (p. 589). These conflicts and the resolutions thereof, Wood et al. found, provided opportunities for discussion and reflection that resulted in the teacher's change in beliefs about mathematics, about learning, and about teaching.

Leatham (2006) describes the process of changing teachers' beliefs as a change in the relative strength of those beliefs:

> One goal of mathematics teacher education, however, might be to affect teachers' beliefs about mathematics such that those beliefs [considered most "desirable" by teacher educators] move high on the list of those beliefs that most influence teaching. In order to have this impact, however, teacher educators and the teachers themselves need to become aware of the beliefs that are currently filling those "most influential" roles. From this perspective, teachers' belief systems are not simply "fixed" through a process of replacing certain beliefs with more desirable beliefs. Rather, teachers' beliefs must be challenged in such a way that "desirable" beliefs are seen by teachers as the most sensible beliefs with which to cohere. (Leatham, 2006, p. 100)

This study examined teachers' changing beliefs. Leatham's description of change as one of change in the relative strengths of desirable beliefs rather than the supplantation of beliefs provided a useful framework for teachers' changing beliefs.

Quality Mathematics Instruction

"Although there is general agreement that quality of mathematics instruction is important, it is generally not directly addressed in most research studies" (Koehler & Grouws, 1992, p. 124). Additionally, "there is no universal agreement on what constitutes 'good mathematics teaching'" (Thompson, 1992, p. 127). Some definitions of quality stem from philosophical visions of teaching, such as Dewey's vision of learning through experience, or constructivist philosophies of teaching and learning. Empirical efforts to define quality mathematics instruction have generally focused on student achievement data and correlations between student achievement and teacher variables such as subject matter preparation (e.g. Monk, 1994), or student achievement and classroom instruction variables such as a focus on higher-order thinking skills (e.g. Wenglinsky, 2002). Other efforts have focused on comparing experts and novice teachers for quality differences in the presentation of lessons (e.g. Borko & Livingston, 1989; Leinhardt, 1989). Studies of student achievement and studies of experts both support the use of strategies that are consistent with constructivist philosophies of teaching and learning.[13] Evidence abounds that constructivist teaching methods provide higher quality instruction and higher student achievement.[14] Given the preponderance of studies, I limit my discussion below to reviews of the literature.

Slavin, Lake, and Groff (2009) used best-evidence synthesis to review the literature on student mathematics achievement with respect to mathematics instruction. They included 100 randomized or matched-control group studies of mathematics curricula, computer-aided instructional programs, and professional development programs aimed at changing mathematics teaching. “In particular, both the elementary review and the current review [of middle and secondary level mathematics programs] find that the programs that produce consistently positive effects on achievement are those that fundamentally change what students do every day in their core mathematics classes” (Slavin, Lake, & Groff, 2009, p. 887). This “fundamental change” involves a shift to the use of strategies such as cooperative grouping, metacognitive strategies, and individualized instruction – all non-traditional methods of instruction.

Ross, McDougall, and Hogaboam-Gray (2002) reviewed 154 studies, both qualitative and quantitative, from the literature on the implementation of reform-

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[13] This categorization of higher quality teaching strategies as constructivist is mine. The researchers and studies cited do not label these strategies traditional or constructivist.

[14] This research does not automatically indicate that traditional methods of instruction are of low quality, but the suggestion is there. Silver et al. (2009) state that “although a direct link has not been established between the generally low levels of student achievement and this portrait of canonical school mathematics teaching practices in the American schools, it is difficult to deny the plausibility of a connection” (p. 503).

oriented[15] mathematics initiatives to examine the effects on student achievement, barriers to implementation, and ways to overcome implementation barriers. They assert that: “the issue is not whether reform in mathematics education contributes to student achievement (it does), but why implementation has been such a rare, fleeting occurrence and what can be done to support teachers’ efforts to change their practice” (p. 123). The literature review found that students in reform classrooms scored better on tests that measured reform values, had better attitudes about math, and did not score worse on traditional tests of math achievement and sometimes scored better. Ross et al. note that the positive effects on student achievement were only visible when there was “substantial” (p. 131) implementation of reform, something that was rare. The barriers to implementation were numerous and included, for example, lack of teacher content knowledge, a mismatch between teachers’ beliefs about mathematics and the reform agenda. Together the Ross et al. review and the Slavin et al. (2009) review find that when constructivist methods are substantially implemented in ways that fundamentally change traditional mathematics instruction, higher student achievement results.

Mathematical Quality of Instruction (MQI). One current description of lesson quality in mathematics is provided by Hill et al. (2008) in their study of the relationship between mathematical knowledge for teaching (MKT) and mathematical quality of instruction (MQI). MQI examines five dimensions of instruction drawn from the research literature documenting both effective teaching methods and ineffective

[15] Recall that the term reform-oriented describes what this dissertation calls constructivist. See discussion of traditional and constructivist philosophies of teaching earlier in this chapter for a rationale. Ross, McDougall, and Hogaboam-Gray agree with the classification of the reform-oriented as constructivist when they write: “although the philosophy behind the [National Council of Teachers of Mathematics’] Standards is appropriately described as constructivist, this label was assigned by Romberg to gain political support after the Standards were written (Bossé, 1995)” (2002, p. 131).

teaching methods. The five dimensions of MQI are (National Center for Teacher Effectiveness, 2012):

- *Richness of the mathematics.* A lesson is rich in mathematics if it (a) includes explanations of the meanings of the mathematics or connections among mathematical ideas or representations, (b) presents multiple solutions, (c) develops generalizations from examples, and (d) uses mathematical language precisely and fluently.
- *Student participation in meaning-making and reasoning.* High quality lessons require students to engage cognitively with the mathematics (e.g. determine mathematical meaning of content, find patterns or connections, justify conclusions, examine claims and counterclaims, or make conjectures) and provide mathematical explanations.
- *Errors and imprecision.* High quality lessons are clear in the presentation of content, use precision of language and notation, and are free of mathematical errors.
- *Working with students and mathematics.* High quality lessons are ones in which teachers respond to students' mathematical ideas and remediate errors thoroughly, with attention to student misconceptions.
- *Classroom work is connected to mathematics.* High quality lessons spend the majority of instructional time on activities that develop mathematical ideas.

Hill et al. (2008) found an association between teacher content knowledge[16] and MQI, but also found that a number of factors influenced this relationship including the nature of teacher beliefs about how mathematics should be learned, teacher beliefs about how curriculum materials should be used, and teacher access to curriculum materials. These findings lend credence to the idea that teachers' beliefs about quality mathematics lessons may well impact the quality of instruction.[17] Initially, this study used MQI to examine teacher beliefs about quality lessons. During the course of analysis, teacher beliefs were found to incorporate beliefs about instruction in areas that were not math-focused. Hence, using primarily MQI to examine teachers' beliefs was limiting.[18]

Teachers' beliefs about quality mathematics lessons. The research provides little insight into what *teachers* believe about quality mathematics lessons (Wilson et al., 2005). Although there are surveys of teacher beliefs about lesson quality (e.g. Banilower et al., 2013), these surveys examine teachers' responses to instructional belief statements written by the survey developers and do not examine teachers' beliefs with enough detail to understand how teachers interpret these beliefs statements, whether teachers' interpretations of the belief statements match the interpretations of the researchers,[19] and how teachers' beliefs may be expressed in their mathematics lessons.

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[16] Teachers' level of content knowledge was measured using the MKT measures, a measure used to examine teachers' mathematical knowledge for teaching (MKT). See Chapter III for a detailed description of the MKT measures.

[17] Other studies have also found that teachers' beliefs impact their instruction. For example, teachers' beliefs may limit the effects of their content knowledge (e.g. Provost, 2103), or change the way they handle and errors in the classroom (Bray, 2011). Further examples are discussed below.

[18] See also Issues with MQI Coding section in Chapter IV for a more in-depth discussion of this issue.

[19] For example, the NSSME (Banilower et al., 2013) includes the belief statement "hands-on activities/manipulatives should be used primarily to reinforce a mathematical idea that the students

Wilson et al. (2005) studied experienced teachers' perceptions of good mathematics teaching and how it develops. The teachers thought that "good teaching requires a sound knowledge of mathematics, promotes mathematical understanding, engages and motivates students, and requires effective management skills" (p. 83). The teachers believed that experience was the primary factor in developing good teaching but that education, personal reading and reflection, and interaction with colleagues also contributed to the development of good teaching. Wilson et al. did not study how the teachers' beliefs were related to content knowledge nor how those beliefs changed.

Teacher Learning

Teachers come to teaching with some knowledge about the profession. They begin this knowledge acquisition as a student, although that knowledge may be skewed due to the fact that it is acquired without full knowledge of the pedagogical decisions that lie behind the activity of teaching (Lortie, 1975/2002; Sugrue, 1997). The knowledge of teaching is further expanded through teacher education programs and student-teaching experiences (Brown & Borko, 1992). Teacher learning does not stop upon entrance into the profession; there is no doubt that teachers continue to learn across their careers (Wilson & Berne, 1999). Teachers learn about content, about teaching, and about students; and they learn these things in a variety of ways.

> Practicing teachers participate in mandatory part-day or day-long workshops sponsored by their school district. They pursue individual learning

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have already learned" (p. 23). In discussing these results, Banilower et al. state: "Similarly, from 39 to 52 percent agree that hands-on activities/manipulatives should be used primarily to reinforce ideas the students have already learned, despite recommendations that these be used to help students develop their initial understanding of key concepts" (p. 22). This statement implies the assumption that the only two ways to interpret the belief statement are that hands-on activities/manipulatives should be used either (1) to help develop initial understandings, or (2) to reinforce concepts that students have already learned. The authors appear not to consider a third possible interpretation: that hands-on activities/manipulatives are not appropriate ways to teach mathematics.

> opportunities: enrolling in Master's courses, signing up for summer and weekend workshops, joining professional organizations. Some learning, no doubt, goes on in the interstices of the workday, in conversations with colleagues, passing glimpses of another teacher's classroom on the way to the photocopying machine, tips swapped in the coffee lounge, not to mention the daily experience of the classroom. (Wilson & Berne, 1999, p. 174)

Much of the research literature on teacher learning is about pre-service teacher learning or that of in-service teachers in their first few years of teaching. "While knowledge about beginning teachers is important," Wood et al. caution, "the manner in which practicing teachers learn and change is also crucial" (1991, p. 589). The discussion below is limited to a basic understanding of the current research on learning as it applies to in-service teacher learning through professional development opportunities, and to the focus of this study: teacher beliefs about quality mathematics lessons.

Professional development can be an effective way to improve teaching. Although the National Mathematics Advisory Panel (2008) report states that little is known about the characteristics of effective professional development in mathematics, Desimone (2009) argues that assertions such as this may be due to "misconceptions about trade-offs of different methods used to study professional development's impacts" (p. 181) and that "there is an empirical research base to support the identification of a core set of features of effective professional development" (p. 181). Desimone's review of qualitative and quantitative research on professional development identifies five core features of effective professional development: content focus; active learning; coherence, defined as consistency with teachers' knowledge and beliefs and consistency with policies at the school, district, or state level; duration; and collective participation, such as of groups of teachers from the same school, grade, or department. This core set of features produces "changes in [teacher] knowledge, practice, and, to a lesser extent, student achievement" (Desimone, 2009, p. 183). The potential effects of professional development on instruction caused Ross et al. (2002) to call professional development "the most powerful mechanism" (p. 132) for increasing implementation of mathematics reform.

In their examination of exemplary professional development opportunities from the research literature, Wilson and Berne (1999) selected two high-quality, well-respected studies in each of three categories to examine for themes. Their categories included opportunities for teachers to: (1) talk about and "do" subject matter, (2) talk about students and learning, and (3) talk about teaching. Each of these areas was found to impact teacher knowledge and/or beliefs. The subject matter category is especially relevant to this study. "Essentially, what these [exemplary] professional development projects appear to be doing when they ask teachers to become scientists or mathematics learners or book club participants is to engage them as learners in the area that their students will learn in but at a level that is more suitable to their own learning" (Wilson & Berne, 1999, p. 194). The professional development used in the training phase of this study engaged teachers in active learning of mathematics content. The purpose of this study, however, was not to gauge the effectiveness of the professional development, but to examine the effect of learning mathematics on teacher beliefs about quality lessons.

One series of seminal studies regarding the connection between professional development and beliefs are the Cognitively Guided Instruction (CGI) studies. The

CGI studies involve professional development in which elementary teachers were trained in the ways in which students learn addition and subtraction. In one of these studies Peterson, Fennema, Carpenter, and Loef (1989) examined the relationship between first- and second-grade teachers' pedagogical content beliefs, pedagogical content knowledge, and student achievement in mathematics. This 1989 study examined teachers' knowledge and beliefs before participating in the CGI professional development training. The study found positive relationships between beliefs, knowledge, and problem-solving finding that teachers with a cognitively based belief system used more problem solving tasks in their teaching and spent time with students developing counting strategies before teaching addition and subtraction facts. The teachers with a cognitively based belief system had more knowledge about the classes of addition and subtraction word problems and used these in their instruction. In addition, students of teachers with cognitively guided belief systems performed better on problem-solving tasks and equally well on computation tasks.

The 1989 CGI study (Peterson et al., 1989) identified the connection between teacher knowledge and beliefs, particularly those about students as learners of mathematics and student achievement. The question remained, however, if training in students' cognitive approaches to problems would result in changed knowledge, beliefs, or student achievement. This question was answered by Fennema et al. (1996) in a longitudinal CGI study. The teachers in the 1996 (Fennema et al., 1996) study participated in a CGI professional development training. During this training, teachers were taught various models of addition and subtraction that students use when solving problems. They were not taught teaching methods, but were taught about students as learners. Over the four years of the study, 18 of the 21 teachers in the study experienced changes in their beliefs and instruction in which teachers changed their role “from demonstrating procedures to helping children build on their mathematical thinking by engaging them in a variety of problem-solving situations and encouraging them to talk about their mathematical thinking” (p. 403). At the same time, these changes in instruction were directly related to gains in student achievement in the areas of problem solving and concepts, with no loss in computational performance.

The 1996 CGI study demonstrates how professional development can change teachers’ knowledge and beliefs which, in turn, can impact student achievement. Especially important for this study is that the CGI training included training aimed at increasing teachers’ knowledge of students as learners in addition to mathematics content, but did not include specific instruction in pedagogy. When teachers in the 1996 CGI study raised pedagogical questions, the researchers encouraged teachers to think about the thinking of their students to make instructional decisions. The researchers admit that the CGI training influenced pedagogy, but “we did not, however, directly prescribe either pedagogy or curriculum for teachers” (Fennema, et al., 1996, p. 409).

Sowder, Philipp, Armstrong, and Schappelle (1998) studied the process of changes in beliefs, mathematics instruction, and student achievement of five fifth- and sixth-grade teachers as they participated in a two-year, twenty-four-session mathematics professional development experience focused on both mathematics content and the ways in which students learn mathematics. In addition, conversations in the professional development often involved pedagogical discussions about the ways in which teachers were presenting information to students in their classrooms. Sowder et al. found connections between the increasing content knowledge of the teachers and their beliefs about mathematics, beliefs about instruction, classroom instructional practices, and student achievement. Of particular interest to this study are the findings regarding teachers' changes in beliefs about instruction over the course of the professional development experience.

Sowder et al. (1998) described the highly individual nature of change, but also described common themes across the cases in their study. First, teachers changed what and how they taught based on their increasing content knowledge. Teachers' instructional goals became more focused on the conceptual understanding of students in addition to procedural skill development. Second, teachers changed their views and use of instructional materials, moving towards a desire to use instructional materials that focused on purposeful development of conceptual understandings and finding that such materials were not always available. Finally, classroom discourse changed and teachers asked students more probing questions to get at student understandings. These changes were accompanied by an increase in student achievement "greater than could be expected in traditional classrooms" (p. 176).

Similar to the 1996 CGI study (Fennema et al., 1996), Sowder et al. (1998) provided professional development that focused both on improving teachers' knowledge of content and knowledge of students as learners. The Sowder et al. study also involved and encouraged discussions of pedagogy throughout the professional development seminars. Both studies saw a change in teacher beliefs about pedagogy with an increase in the belief that instruction should build student conceptual understandings. Both studies also found an increase in student achievement accompanied those changes. In contrast to both the CGI studies and the Sowder et al. study, the professional development used in this study focuses solely on teachers' knowledge of mathematics content, not knowledge of students as learners or pedagogy.

Summary

This chapter examined the research literature with regard to four key areas: teacher knowledge, teacher beliefs, quality mathematics instruction, and teacher learning. This study uses the UTL framework for teacher knowledge that includes knowledge of mathematics content, of pedagogy, and of students. Similarly, teachers hold beliefs about mathematics, about pedagogy, and about students. The quality of mathematics instruction is influenced by teachers' knowledge of mathematics content (e.g. Hill et al., 2008; Peterson et al., 1989) and by teachers' beliefs (e.g. Bray, 2011; Provost, 2013; Raymond, 1997; Thompson, 1984). Teachers continue to learn throughout their careers (e.g. Wilson & Berne, 1999). Professional development with in-service teachers can change teachers' knowledge, beliefs, and instruction, and can impact student achievement (e.g. Desimone, 2009; Fennema et al., 1996; Sowder et al., 1998). Teachers' beliefs about quality lessons have been studied (Wilson et al., 2005), and changes in teachers' pedagogical beliefs in response to professional development have been studied (e.g. Fennema et al., 1996; Sowder et al., 1998). The ways in which teachers define quality teaching in coordination with ways in which teachers' definitions change in response to professional development training in elementary mathematics content only, absent training in knowledge of pedagogy or students, has not.

Chapter III: Research Method and Design

This study examined experienced[20] elementary[21] mathematics[22] teachers' beliefs about quality mathematics lessons and the process by which those beliefs changed in response to teachers' experience in a professional development training in mathematics content.

A Qualitative Case Study using Grounded Theory Methods of Analysis

Case studies are appropriate when the research questions involve "how" a process occurs in situations where context and the phenomenon are closely tied and when the researcher has limited control over behavioral events (Yin, 1994). In this study, each individual teacher represented a "case" because each teacher's experience and the process by which the teacher's beliefs changed was unique. The teacher is the unit of analysis. The use of multiple cases allowed for comparison among participants that both illuminated themes across the individual cases and illuminated variation among the processes of change undergone by participants.

Perspective is critical in examining the type of case study design used. The intent of this study was not to study a professional development opportunity and examine multiple teachers' responses to that professional development. Had that been the intent, a single-case design with multiple units of analysis would have been more appropriate. Rather the intent of this study was to examine multiple teachers' processes of changing beliefs about quality lessons in response to the learning of mathematics

[20] This study defines an experienced teacher as one who has completed three or more years of mathematics teaching.
[21] An elementary teacher is one who teaches in kindergarten through grade eight.
[22] A mathematics teacher is a teacher who spends at least one period of their regular day providing instruction in mathematics, whether or not the teacher provides a grade to that student. Under this definition, many special education teachers fit the definition of mathematics teacher.

content, to gain a holistic view of multiple journeys. A holistic multiple-case design, where each case has one unit of analysis is most appropriate to meet this intent.

Qualitative methods provide depth (Creswell, 2009) and were chosen for the primary data collection of this study. Grounded theory methods were used for data analysis. Creswell (2007) asserts that one may mix approaches "and employ, for example, a grounded theory analysis procedure within case study design" (p. 232). This mix of methodological approaches was chosen for the following reasons. First, the intent of the study was to examine a *process* in-depth. This intent is best met using the approach of case study (Creswell, 2007; Yin, 1994). Grounded theory methods for data analysis allowed for generation of theory describing the change process that was grounded in the experiences of the participants (Creswell, 2007; Charmaz, 2006). Second, grounded theory methodology helped me determine which areas of data were more important to understand the process by which teachers' beliefs changed, and hence where to focus my subsequent data collection and analysis. Not all aspects of my data were equally important. Grounded theory helped me determine a course through the data according to its importance to theory development. Third, the structure and intent of constant comparative analysis, a key component of grounded theory, matches how I think. Therefore, using this approach for data analysis allowed me to use my analytical strengths within a well-recognized qualitative structure.

An Iterative Analytical Process

Constant comparative analysis, a component of grounded theory methods of analysis, requires that the researcher constantly re-analyzes the data according to newly acquired data, to newly determined codes, to categories of codes, and to theoretical models used to understand the data. For readability, this document does not include all of the loops in this iterative process. Many times I coded and re-coded data as I saw something new in it. For example, when I realized that teachers' beliefs had changed in three main areas, I went back to my data and re-categorized my codes to see if all of the changes in teachers' beliefs fit into those three areas. This re-coding, re-categorizing, and re-conceptualizing of the data occurred many times in the course of analysis.

As I analyzed my data, I wrote memos to document my thinking about the data itself and the analysis process. I referred to these memos often and read through them when I got "stuck" in order to re-start my thinking. Conversations with other researchers also stimulated new ways of thinking about the data analysis. It was through one of these conversations that I realized I was studying teacher learning in addition to teacher beliefs. The analysis process was a messy, exciting, sometimes frustrating, but always iterative process – hardly as neat as it appears here.[23]

Research Questions

This study employed the philosophical approach of the case study and grounded theory methods of analysis to answer qualitative research questions. Initially, the study investigated three research questions:

1. What do experienced K-8 teachers state constitutes a "quality mathematics lesson?"

[23] Tracey (2013) discusses the importance of qualitative researchers including the iterations and "mess" of data analysis in their reports. She argues that this inclusion increases the transparency and hence quality of qualitative studies, and also helps future researchers learn about the process of doing qualitative research.

2. When these teachers submit lesson plans for quality mathematics lessons, what instructional goals and strategies do they propose?
3. How do the statements, goals, and strategies change as the teachers gain mathematics content knowledge?

During the course of the analysis the second research question was dropped and the third research question was revised. See Chapter IV for the rationale behind these changes. The final research questions were:

1. What do experienced K-8 teachers believe constitutes a “quality mathematics lesson?”
2. How does the experience of learning mathematics content through inquiry change teachers’ beliefs about what constitutes a “quality mathematics lesson?”

Participants

This study examined the beliefs and content knowledge of eight elementary teachers who taught at least one period of mathematics per day and had completed at least three years of mathematics teaching. The teachers were selected based on their willingness to participate in both the study and in professional development in elementary mathematics content. Teachers who signed up for the professional development used in the training phase were invited to participate in the study.

The initial study design called for eight teachers to be chosen from the pool of respondents in order to provide maximum variation in the demographic information provided by the participants. Maximum variation sampling allows the researcher to “capture and describe the central themes that cut across a great deal of variation” (Patton, 2002, p. 234-235) and “increases the likelihood that the findings will reflect differences or different perspectives" (Creswell, 2007, p. 126). These differences allow for better understanding of the intricacies of how teacher beliefs change and increase the likelihood of disconfirming evidence and cases that challenge theory in addition to those that may support it. Of the fourteen teachers who signed up for the professional development, twelve volunteered to participate in the study and nine fit the inclusion criteria. Of those nine teachers, one had extenuating family circumstances that made full participation in the professional development unlikely. This teacher was excluded from the study and did in fact drop out of the professional development. The eight remaining respondents were chosen for the study. Brief biographical descriptions of each participant can be found in Appendix B.

The participants in this study are all white females. All hold valid teaching certificates in their state and completed a teacher preparation program prior to teaching. Four of the teachers had completed an undergraduate major in an education field (Bethan, Ireane, Leona, and Sue) while the other four teachers majored in non-education fields. Only Alex had a math-related undergraduate major or minor; she minored in statistics. Six of the participants hold Master's degrees, all in fields of education. Only Ireane and Leona do not hold Master's degrees. None of the teachers are National Board Certified.[24] Two teachers entered teaching as a second career: Ellyn did social work before teaching; Marie was in the field of banking.

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[24] National Board Certification is a voluntary teaching credential administered by the National Board of Professional Teaching Standards. For more information see http://www.nbpts.org/

General Education, lead teacher:

- teaches students in their primary mathematics class
- Alex; Marie; Sue

Special Education, lead teacher:

- teaches only students with identified special needs in their primary mathematics class
- Bethan; Ellyn

Mathematics Support (Title I or Special Education):

- teaches students mathematics in an additional class period; students have another general education mathematics class also
- Ireane; Leona; Mary

Figure 4. Participants' Mathematics Teaching Roles.

All participants in the study taught mathematics during their school day, but their roles varied (see Figure 4). Three teachers (Ireane, Leona, and Mary) taught mathematics to students who also had another mathematics class. These teachers provided specific mathematics interventions and extra instruction beyond the students' "regular" mathematics class. These teachers did not provide a mathematics grade to their students, although they did provide instruction. The remaining teachers were lead teachers of at least one mathematics class and were in charge of assigning mathematics grades. Three of these teachers (Alex, Marie, and Sue) taught general education classes while two (Bethan and Ellyn) taught classes made up only of students with identified special needs. All teachers in the study except Leona taught other subjects in addition to mathematics, although Marie's additional subject was a test prep course that involved a great deal of mathematics instruction. See Appendix B for brief biographical descriptions of each teacher. These descriptions contain more specific information on each teacher's roles.

The participants in this study teach and have taught at a variety of grade levels (see Table 2). At the time of the study, all teachers were teaching at least one elementary[25] grade level.

Table 2

Grade Level Teaching Assignments of Participants

	Grade													
Teacher	Pre	K	1	2	3	4	5	6	7	8	9	10	11	12
Alex								P	C	C				
Bethan								C	C	C				
Ellyn							P	C	C	C	P	P	P	P
Ireane							C	C	C	C	P	P	P	P
Leona					C	C	C							
Marie			P	P	P	P	P	C						
Mary			P	P	P	P	C	C						
Sue	C[a]		P	P	P		P	C	C	C	C			

Note. C = currently teaching; P = has taught in the past

[a] In an administrative role at this grade level

Teachers varied in their years of teaching experience and their experience teaching mathematics (see Figure 5). All teachers had completed at least three years of mathematics teaching at the time of their participation in the study. Three teachers (Alex, Ireane, and Leona) have taught math their entire teaching career, while half the

[25] An elementary teacher is one who teaches in kindergarten through grade eight.

teachers in the study have taught mathematics for more than ten years. Only two teachers, Alex and Marie, hold math-specific teaching certifications, both at the middle school level. These two teachers are also the only two teachers in the study who do not hold any Special Education teaching credential[26].

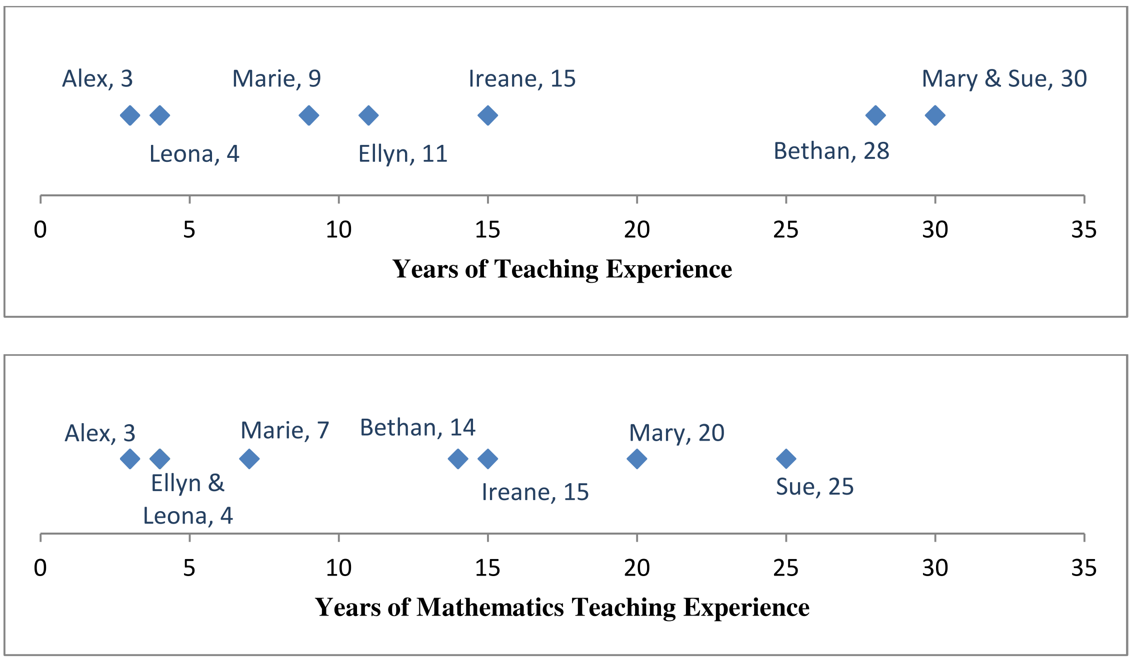

Figure 5. Participants' years of teaching experience.

Teachers' content knowledge of mathematics varied. Difficulties with measuring the mathematics content knowledge of the teachers in the study are discussed in Chapter IV.[27] Although it is not possible, from the data, to rank teachers according to level of content knowledge, it is possible to say which teachers had

[26] Leona holds a General Special Education credential. Sue holds a Special Education teaching credential at the early childhood level. For the purposes of this study, neither teacher is considered a "Special Education Teacher" because their teaching positions require them to act in a role where they have responsibility to all students rather than solely to students identified as having educational disabilities. I used each teacher's job title to help in making this determination.

[27] See The Learning of Mathematics section beginning on page 164.

relatively high and low levels of content knowledge. According to their pre-training MKT scores, Ireane had consistently low content knowledge while Marie had consistently high content knowledge and Alex had consistently very high content knowledge. The other teachers in the study showed variation in their content knowledge. Teachers also expressed varying levels of mathematics ability, according to their own perceptions. These perceptions are discussed in Chapter IV. Teachers in the study varied in their mathematics ability and their own perceptions of their mathematics ability. This variety in the backgrounds, teaching assignments, experience levels, and content knowledge of the participants is important to the multiple case study design and to the development of theory.

Researcher's Role

I am a teacher of grades 6-8 mathematics. I have not taught in any of the same school districts as the participants, nor have I been in a supervisory role over any participants. I have instructed professional development in the region. Although two of the participants were known to me through previous professional development experiences, I was not in an instructor role for any participants during the study and was not the instructor for the professional development used in the training phase of this study.

I coordinated the professional development used in the training phase of this study. The professional development materials and lessons were created by others. I acted as a go-between and secretary for these individuals, coordinating material acquisition and delivery, typing up lessons plans and worksheets as directed by the instructor, and keeping track of administrative tasks such as site facilitation, photocopying, registration, and attendance records. I attended every session of the professional development, opened the classroom, collected lesson plans or journals, handed out study instructions, answered logistical questions, ran errands when materials were missing or glitches happened, and provided technical trouble-shooting.

This study's design included purposeful attention to the possibility that my beliefs might influence the analysis of the data. Qualitative philosophy maintains that all knowledge is subjective and the removal of all influence is not possible. The researcher should be mindful of the ways in which his or her beliefs and prior experiences are impacting the analysis, and that the study design can be used to mitigate such impacts. This study compared teacher statements about quality to pre-established benchmarks of quality from the research literature. I chose the MQI rating scale to examine quality because of its rigor and acceptance in the field. I expected that teachers' beliefs would become more mathematical in nature during the course of the study, and hence score higher on the MQI rating scale. What I did not expect, was how these benchmarks of quality were only one small piece of teachers' beliefs about quality mathematics lessons. The study design called for multiple sources of data, participant verification, careful ordering of data analysis and literature searches, and the deliberate search for disconfirming evidence in order to limit the influence of the researcher's beliefs on the data analysis.

Settings

Teacher beliefs, by their very nature, reside within the individual. The process by which teacher beliefs change is also an internal process. Thus, the setting for each case in this study is situated inside each participant. Data was collected in order to make those mental processes explicit. Data collection and training for this study took place in the northeastern United States. Much of the data consisted of written documentation provided by participants. This documentation was created at any location chosen by the participant. Other data was created at the collection location. Table 3 summarizes the settings for each type of data created and collected. A detailed description of each setting is described in the Data Collection and Ongoing Analysis section below.

Table 3

Settings for Data Collection

Data	Setting(s)
Lesson Plans with Written Reflections	Teacher choice of setting.
Interviews	Mutually agreed-upon location.
Journals	Teacher choice of setting (journals 1 & 2) and professional development classroom or computer lab (journals 2 & 3).
Within-case Participant Verification	Teacher choice of setting.
Professional Development	University conference room and university computer lab.
Cross-case Participant Verification	Teacher choice of setting; teachers logged into an online forum from a location of their choosing.

Data Collection and Ongoing Analysis

Data collection and analysis consisted of three phases: pre-training, training, and post-training. A graphic representation of the data collection and analysis phases is in Appendix C. Data was collected from multiple sources during all three phases of the study. Pre-training data included demographic information, teacher-submitted lesson plans of self-identified quality mathematics lessons, written reflections on those lesson plans, interviews regarding teachers' views on quality lessons and submitted lessons, and participant verification of within-case[28] analysis. Data collected during the training phase included pre- and post-assessments of teachers' mathematical knowledge for teaching (MKT), lesson plans, and journal entries in which teachers reflected on lesson quality and how their beliefs about quality were or were not changing as they participated in the professional development. Post-training data included an additional lesson plan of a quality lesson, written reflections on the lesson plan, interviews, participant verification of within-case analyses, and posts from an online focus group used as participant verification of the cross-case[29] analysis.

Analysis was ongoing and each bit of data was analyzed as it was collected, with the exception of the MKT measures post-assessments which were analyzed only after participant verification of the within-case analyses were complete. Initially, coding and analysis was done by hand. Later, NVivo™ 10 software from QSR International was used to store, code, and organize data. Data analysis was facilitated through this consolidation of the various types of data into one location and the use of

[28] Within-case analysis refers to analysis that uses data collected from or about an individual teacher to find themes in the data collected from that one individual.
[29] Cross-case analysis refers to analysis that uses data from all participants to find themes across data collected from multiple participants.

the software's powerful coding, sorting, and organizing tools. Each individual teacher was considered one case in the case study and data was analyzed within the case for themes. This within-case analysis was coupled with a cross-case analysis of the themes that appeared in multiple cases. Both within- and cross-case analyses were performed for each phase of the study.

Pre-training phase. The professional development used in the training phase of this study was advertised through local email mailing lists and professional groups (see Appendix D for advertising flyer). Interested teachers accessed a website to receive more information and register for the professional development. During registration, teachers submitted demographic information. Teachers who registered were contacted via email in order to set up a meeting regarding participation in the study. Twelve registrants agreed to participate in the study. I met with these teachers individually to sign consent forms (see Appendix E), determine eligibility, and verify and add to previously submitted demographic information. Nine teachers fit the inclusion criteria for the study. Of those nine teachers, one shared family issues that made full participation in the study unlikely. The eight remaining teachers were chosen for participation in the study.

Study participants chose a lesson that they had used with their students and that they considered a "quality lesson" (see Appendix F for selection directions). Teachers submitted a lesson plan via email along with any supporting documentation.[30] The format of submitted lesson plans was intentionally left open so that the required format of the lesson plan did not influence what teachers submitted. Each lesson plan was

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[30] Supporting documentation included worksheets, copies of applicable pages in their text, and, in some instances, teacher-designed tools such as number line cards.

accompanied by a reflection in which the teacher explained why they had chosen the lesson to submit and what made it an example of a high quality lesson (see Appendix F). These lessons and reflections were coded according to emergent themes regarding lesson quality. In addition to coding based on emergent themes, the data was coded according to the five dimensions of MQI. The first round of analysis required quick turn-around time. Teachers submitted lesson plans and within a few days participated in their first interview regarding their beliefs about quality lessons. Teachers' reflections on their lessons were very brief and provided little insight into teacher beliefs. This data was initially most meaningful as starting point for the interviews. When analyzed in conjunction with other submitted data, this data set became more meaningful.

After submitting the lesson plan and reflection, teachers were interviewed regarding their lesson, their written responses, and their beliefs about what makes quality mathematics lessons. Interviews were conducted at a meeting place suggested by and convenient to the teacher. Most interviews were conducted in the teacher's home or classroom. In one instance the first interview was conducted at a coffee shop. This location had a significant amount of background noise and made transcription from the audio recording difficult, although possible. In two instances the first interview was conducted via Skype.[31] The participants accessed Skype from home and I was either at home or in a private study area in the university library. There were no technical issues with these interviews but I felt that it was difficult to observe the meaning of body language during the conversation. As a result of these difficulties, all

[31] Skype is a web-based telephone service that allows for video-conferencing. Both interviews were conducted using the video feature. See www.skype.com for more information.

second interviews were conducted at the homes or work locations of the participating teachers. Interviews were semi-structured to allow for clarification and expansion of teachers' responses on their written reflections. The interviews followed an interview guide model[32] (Patton, 2002) to keep the tone conversational but still cover the specified topics (see Appendix G for interview protocol and questions). Each interview lasted between 29 and 54 minutes, with the typical interview lasting approximately 35-40 minutes.

I used a Livescribe Sky smartpen[33] to audiotape the interviews and take field notes including nonverbal responses and relevant body language during interviews. All interviews were also audiotaped with an additional recording device as backup and transcribed by a professional transcription company.[34] The transcription company returned the transcripts as word processing files. I then went through the documents and edited them for accuracy. In most cases, inaccuracies in the transcripts were due to the teachers' use of education-specific terminology or acronyms. Other inaccuracies were the result of the teacher and me talking over each other or background noise that made the conversation harder to follow. In nearly all instances, I was able to listen to the tape and determine what was being said. While editing the transcripts, I also removed any instances where the participants' real name was unintentionally used in the interview, either by me or by the participant themselves.

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[32] The interview guide as described by Patton (2002) outlines the topics to be covered during an interview but does not limit the interviewer to particular wording or order of the questions for the interview.

[33] The Livescribe Sky smartpen records audio and matches the audio to written notes so that one can match the time at which notes were made to a specific time in the recording. This made it possible to write the word "shrugs," for example, and match that to the point in the audio at which the respondent shrugged. See www.livescribe.com for more information.

[34] Two transcription companies were used during the course of the study: New England Transcription Services of Boston, Inc.; and Daily Transcription.

Teachers' first interviews were analyzed by hand according to previously discovered and emergent themes and, coupled with the reflections on their first lesson plans, were translated into belief statements. To create these belief statements, I organized teacher statements into groups according to topic. I then re-stated these beliefs in one document, including each belief that the teacher expressed. The statements were written to preserve much of the language used by the teachers to describe their beliefs, often including quotes from the interviews or written lesson plan reflections. Once these belief statements were completed, they were emailed to the participants for verification before beginning the professional development. In seven of the eight cases these verification statements were sent out within one week of the interview; the eighth teacher (Mary) received her verification nine days after her interview. Participants verified these statements on their own time and in a location of their choosing (see Appendix H for directions to participants). Three teachers responded to the verification requests via email before the professional development began, one turned in her verification the morning of the first class, and the remaining four verified their beliefs statements during the morning of Day 1 before instruction began but after the pre-test.

Training phase. During the training phase of the study, teachers participated in a professional development[35] experience aimed at improving their mathematics content knowledge of K-8 level mathematics. Multiple sources of data were used during the training phase including pre- and post-assessments of teachers' mathematical

[35] The term "professional development" is used in the field of education to mean a training intended for practicing teachers. Teachers must renew their teaching credentials and states mandate a certain amount of professional development be completed in order satisfy requirements for renewal.

knowledge for teaching using the MKT measures; and teacher-submitted journals, lesson plans, and associated reflections.

The professional development training. The training phase of the study consisted of a professional development commissioned by the researcher using curriculum materials designated by a third party doctorate in the field of mathematics education and instructed by a hired instructor recognized by the state level mathematics teachers organization for her mathematics teaching ability. The professional development, titled Understanding Elementary Mathematics[36] (UEM), covered selected topics outlined by the Common Core State Standards as elementary mathematics content (see Appendix I) and was delivered over the course of six weeks in February and March of 2013. The UEM advertising website described the professional development as follows:

> Understanding Elementary Mathematics (UEM) is a 35-hour professional development opportunity for K-8 teachers. This program will help teachers gain greater knowledge of the mathematics used in the Common Core Standards. Over the course of the 35 hour program, teachers will: examine numbers and operations topics used in the Common Core State Standards (CCSS) for Mathematics, and develop a deeper understanding of the mathematics they teach. This is a course in mathematics content. Teachers will dig deeply into the mathematical content to come away with a greater understanding of the whys of mathematics, not just the hows. In order to dig deeply into the mathematics, this professional development program focuses on the numbers and operations strand and does not cover algebra, geometry, or statistical topics in depth. Nevertheless, numbers and operations have necessary connections to these topics which naturally arise during the professional development. (http://anngaffney.weebly.com/understanding-elementary-mathematics.html)

The professional development consisted of eight class days. Days 1, 5, and 8 were Sundays with sessions from 9:00 am to 4:00 pm (3:30 on Day 5), while the remaining

[36] Copyright 2013 by Gaffney Educational Consulting, LLC. Used with permission.

class sessions were on Wednesday evenings from 5:30 pm to 9:00 pm. Teachers completed the MKT measures on the first and last class days. Actual lesson plans used during the professional development are not in this document in order to protect the intellectual property rights of the authors. Appendix J contains a breakdown of class time and content covered.

UEM took place in a long, narrow room that had the form factor of a conference room. The room was equipped with movable tables and padded chairs on wheels. Each table seated two individuals comfortably. The classroom was arranged so that two of these tables were put together to form one group table that would seat four teachers, two on each side facing each other. Teachers sat in groups of three or four at these tables. Teachers chose their groups for the first class session and the instructor assigned groups for the remaining class sessions. These table groups were placed along the two long edges of the room with an aisle between them. The room was equipped with a whiteboard at one narrow end of the room and a projection station that allowed the instructor to project from a computer to a pull-down screen located so that it covered the whiteboard when in use. During many class sessions, a table was set up in the center of this aisle with a document camera that could project teacher or instructor work onto the screen. This table cramped the center aisle and made instructor circulation more difficult, although still possible. The form factor of the classroom also meant that teachers at back tables had a harder time seeing the screen. The instructor enlarged items on the screen upon teacher request.

Twice during the professional development the teachers used a computer lab. The lab was arranged with rows of individual computers facing the front of the room.

The lab was used on the first and last days of the professional development when the teachers took the MKT assessments and wrote their final reflection. When the computers were no longer needed, teachers returned to the classroom.

UEM used very specific teaching practices in order to deliver content. Each work session began with a warm-up problem, introduced an optional challenge problem for teachers to work on if they finished the assigned activity early, and then posed problems for teachers to work through and discuss at their tables. Teachers were expected to participate in solving problems, in discussing the problems with others, in sharing strategies used to find solutions to the problems, and in an examination of the standards for mathematical practice that they had used to solve the problems. The instructor guided discussions, asked questions to elicit teacher responses, and suggested alternative ways of thinking about problems. In short, UEM was based on constructivist philosophies of learning and guided teachers in using methods of inquiry to discover mathematical ideas and build mathematical understandings.

Videotapes for MQI ratings. Four of 17 work sessions of UEM were videotaped in order to rate the quality of the delivery of UEM lessons according to the MQI rating scale. The collection of this data was not in the original study design. I added it later thinking that it would be an objective way to describe the professional development training and that the changes in MQI characteristics of lesson plans could be matched to the MQI ratings of the professional development. In this way, it would be possible to see if there were similarities in the changes in teachers' lesson plans and the characteristics of the instruction of the professional development training. IRB permission was sought and granted for use of these videotapes only for the purpose of rating the professional development using the MQI rating scale and for no other purpose.

The actual rating of these videotapes using the MQI scale was not completed as part of this dissertation. In analyzing the lesson plans and other data that teachers submitted it became clear that the MQI scale was not a useful way to discuss teachers' beliefs. First, the MQI dimensions are all within the area of mathematics, and teachers' beliefs about quality math lessons were situated in much broader domains. Second, the use of the MQI dimensions as a checklist rather than a rating scale limited the effectiveness of the codes. Third, teachers described lesson plans as being very different than lessons. These issues with the use of the MQI rating scale to examine teacher data are discussed in more depth in the Issues with MQI coding section of Chapter IV. Because the MQI rating scale was not used to examine teachers' beliefs about quality lessons, analyzing the videotapes of the training using this scale would use resources while providing no added value. For these reasons, this data was not analyzed.

The MKT measures. The professional development experience included pre- and post-assessments of teachers' mathematical knowledge for teaching using the MKT measures. The MKT measures used in this study were devised as part of the Study of Instructional Improvement at the University of Michigan (Ball et al., 2005; Hill, Schilling, & Ball, 2004). These measures were specifically developed to measure mathematical knowledge for teaching and are well-suited for use when "examining how knowledge of mathematics for teaching develops, as in professional development opportunities that focus on broad areas of mathematics and strive to improve teachers' mathematical reasoning and analysis skills" (Study of Instructional Improvement, n.d.). The measures used in this study are Elementary Number Concepts and Operations, 2008, Forms A and B (Learning Mathematics for Teaching, 2008). These measures are normed for teachers who teach mathematics in grades kindergarten through eight and are not well suited to highly mathematically knowledgeable individuals as they do exhibit a ceiling effect. Elementary Number Concepts and Operations, 2008, Forms A and B have reliabilities of 0.84 and 0.85, respectively, using a two-parameter model, or 0.81 and 0.83 using a one-parameter model (Study of Instructional Improvement, n.d.) and a standard error of measurement (SEM) of 0.43175 (H. Hill. personal communication, November 19, 2013).

Teachers completed both pre- and post-assessments using the Teacher Knowledge Assessment System (TKAS), an online administration of the MKT measures coordinated by the Learning Mathematics for Teaching Project, the original developers of the MKT measures. The TKAS system assigned teachers either Form A or Form B as a pre-test and used the other form as a post-test. Teacher's raw scores were converted to Item Response Theory (IRT) scores by TKAS. These IRT scores are a way to correlate each teacher's performance with a number of standard deviations away from the mean of the norming group. IRT scores are linear and take into account difficulty differences in the two forms of the assessment. Although pre-assessment data was examined as it became available, post-assessment data was not analyzed until after the within-case participant verification had been completed during the post-training phase of the study. At that time the SEM was used to create error bars for each individual teacher's score. These scores were then examined to see if the changes in teachers' scores were outside the standard error.

Qualitative data and analysis. During the training phase, teachers kept a journal of their thoughts about the attributes of quality mathematics lessons and the ways their beliefs were evolving or were reinforced (see Appendix K). Teachers were encouraged to write as often as they would like, although all but one (Sue) chose to only write the three required journal entries. The journal entries were roughly timed to cover each third of the professional development. As part of their participation in the professional development, teachers completed a mid-way and final reflection. Participants had the option of submitting these reflections as part of their second and third journals. Some teachers chose to submit their midway reflection as part of their second journal and all participants chose to submit their final reflection as part of their final journal entry. The final reflection was completed in the computer lab on the last day of the professional development. This may have impacted the data collection in that the teachers were required to spend a full hour on the reflection or combined reflection/journal. It is unclear if teachers would have spent more time on these reflections/journal entries had they completed them on their own time and in a location of their choosing. Teachers were allowed to continue or add to these reflections and journals after the class time. Two participants chose to do so.

In addition, teachers submitted a lesson plan and associated reflection approximately two-thirds of the way through the professional development experience (see Appendix F). These lesson plans and reflections were similar to those collected in the pre-training phase but also asked teachers to reflect on how their second lesson plan compared in quality to their first lesson plan. Journal entries, lesson plans, and reflections were coded and, coupled with MKT pre-assessment data, analyzed for themes both within and across case.

Post-training phase. After completing the professional development, teachers submitted a third lesson plan and written reflection for a lesson that they considered to be of high quality. All data was coded. These procedures mirrored the pre-training and training phases. Teacher-specific interview questions were devised based on the data collected thus far. Teachers were interviewed following the same semi-structured, interview guide model as in the initial interview, with the addition of questions relating to changes in their beliefs on quality lessons (see Appendix G) and the specific questions created for each teacher. For example, in her third journal Ireane wrote about how she used to think that "simply knowing algorithms would help me understand math. It was my experience in Understanding Elementary Mathematics that taught me how seeing mathematics is truly the way to learn, understand, and ultimately think" (IC, Journal 3-2, p. 1).[37] I wanted Ireane to talk more about this "seeing" of mathematics. In her interview, I read her this section of her journal and asked her to elaborate. Other teacher-specific interview questions asked teachers clarifying questions about the ways in which their lesson plans were implemented or asked teachers to share what happened when they used those lessons with students.

These interviews were lengthier than the initial interviews, lasting from 34 to 69 minutes with the typical interview lasting between 45 minutes and one hour. As before, interviews were recorded using the Livescribe Sky smartpen to match the

[37] Each quote from the data is cited with the initials of the participant's pseudonym, the piece of data (e.g. Lesson Plan 1, Journal 3, Interview 2, etc.), and the page number. Page numbers for interview data refer to the page number of the transcript.

recordings to field notes, audiotaped using a backup system, transcribed, and examined for themes. Statements of beliefs were created for each teacher outlining both their new beliefs about quality lessons and the process of how their beliefs changed. The procedures for these statements mirrored those used in the pre-training phase. Particular attention was paid to staying with-in case for this analysis. Participant verification was used to verify and/or modify these within-case results using similar procedures as in the pre-training phase (see Appendix L).

MKT post-assessment data was examined and matched to each case only after the within-case analysis had been verified by participants. The sequencing of the analysis of the results of the MKT measures helped to ensure that teachers' changes in beliefs were understood before attempts were made to correlate them with changes in content knowledge. Teachers' scores were analyzed using the standard error of measurement to examine individual performance.

Further analysis using data sorting and refining. After the within-case beliefs were verified, extensive cross-case analysis was performed to determine common themes, the underlying meaning of themes, and a theory of the setting. It was at this point that hand-coding and analyzing the data became unwieldy. I entered all data into NVivo™ 10, a qualitative research software package by QSR International. This software package allowed for coding and matching of data so that, for example, references to a lesson plan in written reflections could be linked to that portion of the lesson plan. It allowed for quick searches to be undertaken compiling all data that had been coded a certain way, and complex searches for all data coded one way and not another way. This simplified the organizational aspects of data analysis.

I began this more structured analysis following the recommendations of Kathy Charmaz in *Constructing Grounded Theory* (2006). I began by coding each incident of all qualitative data according to the idea the teacher expressed and then sorted, combined, and separated codes to better describe the differences and similarities within each code in a process that Tracy (2013) calls “lumping and fracturing” (p. 190). This analysis in essence validated previous findings but also made it possible to refine the categories and organize data more efficiently. I first combined codes that were essentially different wordings of the same idea. I was careful, in this re-coding, to make sure that the change in code was not changing the teacher’s original statement. After combining terms that were essentially different wordings of the same idea, I began separating categories to clarify the boundaries between them. For example, the code “building confidence” was further separated to include the teacher’s method for building confidence (when stated), such as “building confidence by trying it” and “building confidence by showing how much students already know.” This fracturing process was not illuminating themes; I was seeing the trees and not the forest. I changed my focus and began grouping my codes in a process that Tracy (2013) calls lumping. In doing so, I found I was grouping them into areas that included a focus on the mathematics content, on the pedagogy, or on students as learners. When these groupings were complete, I compared them to the areas where teachers had identified changes in their beliefs, content knowledge, or practice. Finally, I examined teachers’ demographic information and teaching role to see if any of the trends I had noticed were related to these characteristics.

The findings from this analysis including the theory of the setting were shared with participants in an online focus group used to verify the results (see Appendix M for directions and Appendix N for focus group questions). An online focus group was chosen because online interactions may contribute to more reflectivity in the postings (Stiler & Philleo, 2003; Tracy, 2013) and participants may be more willing to share ideas that may be perceived as negative (Schrum, Burbank, & Capps, 2007; Tracy, 2013). In addition, the online nature of the interactions allowed for the individual participants to retain their anonymity to each other, to have varying schedules, and still to collaborate on the process. This focus group used the comment capabilities of an interactive website created using Google Sites. [38] This allowed teachers to comment on the results as reported and to discuss each others' comments. Focus group data was entered into NVivo™, coded, analyzed and used to make revisions to the findings. Participants chose their own setting for participation in the focus group and logged into the website from a location and at a time of their choosing. Participant comments were downloaded, coded, and used to re-examine the analysis.

Ethical Considerations

The study design included safeguards to ensure that participants' identities remained concealed and that the data was only used in ways allowed by the participants. Approval was obtained from the Rivier University Research Review Board before the onset of data collection and additional permission was sought when data collection methods changed, as with the videotaping of the professional development training.

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[38] Google Sites allowed me to create my own website, limit comment capability and access to only study participants, and protect the confidentiality of participants. An added bonus is that this was a free service. The web address for this site is http://sites.google.com/site/qualitymathlessonsstudy/ .

All registrants for the Understanding Elementary Mathematics professional development training were allowed to participate in the training free of charge, regardless of whether or not they qualified or chose to participate in the study. Participants did not receive compensation for their participation in the study, but did receive a certificate of participation stating that they participated in 15 hours of professional development in addition to the certificates of participation distributed in the UEM training. This certificate of participation documented the amount of time the participants spent participating in the study and did not carry any value other than documentation of time spent on a professional activity.

All participants chose a pseudonym when signing consent forms for participation. Although I knew which teacher corresponded to each pseudonym, all documentation and data submitted was labeled with the teacher's pseudonym, not their real name. When data was transmitted via email, I removed email addresses or any other identifying information from the correspondence, and labeled the data with the pseudonym. At the conclusion of the study I deleted all email correspondence related to the study from my email inbox and storage folders. Similarly, the paperwork matching real names to pseudonyms was shredded at the end of the study.

When videotaped data was added to the study, permission was only sought and granted for the analysis of those videotapes using the MQI rating scale. Consent was obtained from all teachers participating in the professional development and from the instructor of UEM. This data has not yet been analyzed. The digital video files are stored on my password protected computer and a locked back-up drive. In order to protect the identity of all teachers taking the professional development, the videos will only be shown to the MQI raters. The decision was made to delete these videos after use and not retain them for future research because I wanted the teachers to feel comfortable and not worry that they may be exposing their mathematical inadequacies in a way that may be traced back to them. When rating occurs, each rater will be given a DVD of the files. These DVD copies of the videos will be destroyed after rating and the original digital video files will be deleted.

All data, with the exception of the identifying information discussed above, is retained for future research. Participants received some results of the study, the results that relate to their data, during the participant verification process. In addition, participants were informed that they may request a summary of results at the completion of the study.

Issues of Validation and Reliability

Qualitative researchers have used various terms to address issues of quality in social science research, such as Lincoln & Guba's (1985) "trustworthiness." I have chosen to use the more traditional terms in order to speak more clearly to the concerns of positivist readers. Yin (1994) groups issues of quality into four main tests: construct validity, internal validity, external validity, and reliability. Each of these tests was considered in the design of this study and analysis of the results.

Construct validity. Construct validity examines whether the data collected actually examines the phenomenon to be studied. In this study, construct validity addresses whether or not the teachers' statements of beliefs collected via lesson plans, reflections, and interviews actually reflect the participants' beliefs about what constitutes a "quality mathematics lesson" and the ways in which those beliefs changed. The study design included multiple sources of data and participant verification to enhance construct validity. In addition, a chain of evidence was preserved in the dissertation so that each teacher's statement is tracked to the piece of data from which it came. These features help to enhance construct validity.

The study does not examine teacher actions in the classroom to determine if those actions are consistent with teachers' expressed beliefs. Kagan (1990) suggests that, due to inconsistencies in expressed and stated beliefs, it would be wise to do this, presumably to increase construct validity. Other researchers assert that teachers' inconsistent beliefs and actions can be viewed as consistent when the contexts are considered (Beswick, 2005; Ernest, 1989; Skott, 2001). I did not wish to examine teachers' classrooms because the contexts provide barriers to teachers' implementations of their beliefs. I wanted to study their ideals, recognizing that these ideals may not be implemented in the context of their classrooms. I argue that adding this additional source of data for teacher beliefs would not increase construct validity because the construct being studied is the teachers' espoused beliefs, not their enacted ones.

Internal validity. Internal validity examines issues of causality. In this study, issues of internal validity are raised when determining if teachers' changes in beliefs about what constitutes a "quality mathematics lesson" are a result of their experience learning conceptual mathematics content through inquiry or are a result of some other phenomenon. This study set out to examine the relationship between increased knowledge of mathematics content and changing beliefs about lesson quality. The study design included the use of the MKT assessment and teachers' own statements examine this potential causal relationship. The data collected was unable to determine a causal relationship between a gain in content knowledge and changing beliefs about quality lessons. The data did present a rival theory: teachers' experience learning mathematics through inquiry was the primary influence on changes in their beliefs about quality lessons. The original theory may also hold merit, but its merit could not be determined from the data collected in this study.

External validity. External validity refers to the extent to which one may generalize these results and have them still accurately depict a population. Because of the multiple-case design, this case study can be thought of as individual single-case studies replicated eight times. In an ideal multiple-case design, each of the eight participants would have been chosen purposefully to illuminate particular aspects of the theory. Although the original study design called for maximum variation sampling in order to examine the variety of experiences and processes of change, the study was actually conducted with a convenience sample due to the small number of willing and eligible participants. This does limit this study's external validity. Despite this, the eight teachers in this study represented many types of individuals with varied backgrounds, teaching experiences, and learning experiences. The variation in the participants was quite high. In addition, the cases in this study were examined following a multiple-case study design; each case was examined on its own with reports on each individual case detailing how the teacher's beliefs had changed. These individual cases were examined for similarities and differences in change processes in a comprehensive cross-case analysis. The design feature preserves the multiple-case aspect of the study and increases external validity. External validity is decreased, however, by the fact that these eight case studies were conducted at the same time, not one after another.

This study asserts that the process of learning mathematics through inquiry changed *experienced* teachers' beliefs about quality mathematics lessons. The study used multiple, varied cases to examine this assertion. All cases used the same intervention and took place at the same time. It is possible that some unique feature of the particular intervention, other than the methods of inquiry used, is responsible for these changes. This study should be replicated with other interventions, including those using constructivist methods and those using more traditional methods, to examine this assertion in more detail. The body of literature would benefit from a direct examination of the ways in which various types of learning experiences impact experienced teachers' beliefs about quality mathematics lessons.

Interestingly, this study's findings regarding teacher beliefs were essentially identical to Wood et al.'s (1991) examination of an experienced teacher in her classroom nearly twenty-five years before this study took place. This congruence speaks to external validity. Despite the difference in the type of learning experience for the teacher, the location, the year of observation, and the duration of time over which data was collected, the similarities are so great that, taken together, these studies enhance one another's findings.

Reliability. A study is considered reliable if it can be repeated with the *same cases* with the same result (Yin, 1994). This does not mean that if the study were performed again with other individuals or other cases that the results would be the same, but rather that other researchers performing the same study in the same way with the same participants would end up with the same results. Yin (1994) suggests explicitly stating the procedures used in a study and presenting alternate theories as ways to increase reliability. This dissertation presents not only the data collection procedures, but also the data analysis procedures to increase reliability. In addition, rival theories were explored. In one instance, that of causality, the rival theory better matched the data and was presented as a finding.

Chapter IV: Data and Data Analysis

This multiple-case study examined eight teachers' beliefs about quality mathematics lessons. Each of the eight teachers in the study was considered as a single case. Data was analyzed within each case and then across cases both before and after the professional development. The use of both within- and cross-case analysis allowed for the examination of the data from the perspective of the individual and of the teachers as a whole.

Eight Case Studies

In this multiple-case study design, data was collected from eight individual teachers. Each teacher was examined as a single case with the individual as the unit of analysis. The following cases provide descriptions of each teacher's journey. Each individual experience provides insight into the change process and is noteworthy in its own right. The eight cases are examined below in order of the teachers' years of experience, beginning with the most experienced teachers. Pseudonyms have been used throughout this study to protect the identities of the teachers involved.

Sue. Sue Daniels was a veteran teacher with 30 years of teaching experience at the time of the study, 25 of those years teaching mathematics. She had experience teaching in elementary classrooms and as a preschool coordinator in addition to her position teaching sixth through ninth grade mathematics in a virtual environment. At the time of the study, Sue had been teaching these virtual mathematics courses for 5 years. Sue majored in elementary education in college where she completed a teacher preparation program and earned an elementary teaching credential. Sue completed a Master's degree in curriculum and instruction with a focus on computer education and has demonstrated competence in middle level mathematics content through her scores on the PRAXIS[39] exam. Teaching was Sue's first career.

Sue's math teaching was done in a virtual environment. Students completed lessons on the computer and interacted with Sue online and via the telephone. Sue monitored their progress, discussed their work with them, addressed errors, and assessed their knowledge through their work on the computer and via their telephone conversations. One way Sue assessed student knowledge was through the use of what she called "discussion-based assessments" after each unit test. In a discussion-based assessment question, Sue asked students to explain or describe something about the mathematics. Sue said this helped her to make sure that the student, rather than someone else (e.g. a parent), was doing and understanding the work. Sue had limited control over the initial lessons students completed, but had a great deal of freedom and flexibility in how she addressed student concerns, errors, or issues.

Sue was a proponent of online learning and described the experience for students as "like having a teacher in the room with them" (SD, Interview 1, p. 2). She noted that when students work through their online lesson it appears one part at a time, similar to the way in which a teacher in the classroom would present material. She contrasted this to the way a homeschooler learning from a textbook would see the whole lesson all at once. She believed the material appearing one part at a time provided better learning opportunities. Sue liked that students in her online classes could replay the videos if they wanted to, unlike students in a traditional classroom environment. Sue added that this method of instruction allowed for individual pacing

[39] The PRAXIS is a series of exams administered by the Educational Testing Service. Many states use PRAXIS scores as one aspect of determining eligibility for certification. See www.ets.org/praxis for more information.

which, she said, is important for those who have chronic illnesses or are athletes. She liked that students could "just do it in order and keep the pace" (SD, Interview 1, p. 6) according to their schedule. This helped students avoid having to "play catch-up," (SD, Interview 1, p. 6) the way students in a traditional classroom setting would, when they missed lessons. The online environment, Sue believed, was the best of all worlds.

Sue described her role in the learning process in this virtual environment: "The piece where I come in is the student who gets overwhelmed or misses a concept, then I come in and get them back on track and they haven't practiced a lesson wrong ten more times. We can stop it right there" (SD, Interview 1, p. 2). Sue valued this immediate feedback. She described herself as being "good at finding where students make their mistakes, even if it's in a different context, different way" (SD, Interview 1, p. 8). This strength helped Sue help her students.

Sue believed that students benefit from hearing the "thinking" (SD, Interview 1, p. 10) of the teacher to learn methods and strategies for solving problems. She said she often shows students how to solve a problem by showing all of the steps as a think aloud. "Some of the bright kids have done it in their heads for so long that they don't know how to use pencil," she said in her first interview. "As math gets harder you've got to be using paper and pencil" (SD, Interview 1, p. 10). Sue believed that showing her own thinking "gives them [students] a strategy" (SD, Interview 1, p. 10) to use to solve the problem.

Sue believed lessons should present only one rule at a time so that students could work with that rule and "understand"[40] (SD, Interview 1, p.10) it before moving

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[40] Context clues suggest that Sue used the word "understand" in its instrumental sense (Skemp, 1976) and means that students have learned how and when to correctly apply a rule.

on to the next rule. These rules should be practiced, learned, and used. Sue believed quality lessons should have mini-practice sessions throughout where the student learns only one or two rules before practicing them. Similarly, Sue said in her first interview that “homework in math is very, very important. You have to practice it. You can’t just learn it in the classroom without follow-up on your own and get it right” (SD, Interview 1, p. 7). To this end, Sue believed that it was important for lessons to show students common errors so that the students are less likely to make those mistakes themselves.

Sue firmly believed that the mathematics students are learning must relate to real life. She felt that the students needed to know why they are learning the particular mathematical skill and that connections to their real world are an important part of this. She said that these connections should be in the textbook so that parents are also able to see why the content is important. When teaching the Pythagorean Theorem, for example, Sue said she uses an example of a ladder leaning up against the house and determining if it can reach the window or not. When Sue taught in a traditionally formatted fifth grade classroom, she said she tried to purchase units that had a real world component. Sue gave an example of a unit where the students in the class made a mock taco restaurant. She described the unit as bringing the mathematics of art and cooking into the classroom in addition to the traditional math topics.

Sue’s descriptions of her own classroom practices match very closely with the beliefs of a traditional mathematics teacher. As the study progressed, Sue continued to come back to the theme expressed in her pre-training data about learning and practicing procedures one skill at a time with immediate feedback as to whether or not the child was accurately using that procedure. In her third journal entry, Sue wrote: "I would define a high quality lesson as one that stays with one new skill throughout the lesson, shows why the math works, not just the rules for simplicity, and relates the math to real world practicality. Students would be engaged in the math, even if on paper, and not just watching it on an overhead projector screen. They would be able to generalize concepts from math to science to home and job" (SD, Journal 3, p. 1). In the final interview Sue added that a high quality lesson is a lesson

> that teaches the concepts, practice the concepts, but you also manipulate and learn why it works. It's the why. It's not how. Because that comes later. The kids have to know why, and I do believe it has to be related to the real world. And in math precision is one of the last…, but you do have to be right. It's not like reading a poem where you can like interpret. There's no interpretation. It's either eight or it's not eight. (SD, Interview 2, p. 25)

Sue was beginning, at the end of the study, to broaden her view of quality mathematics to include more than procedures, but she still valued procedural fluency and accuracy above conceptual understanding. This is evident in Sue's example, from her second interview, of why students need to understand procedures:

> Like say for instance… the [word problems] like the farmer's got so many horses and so many ducks, and this is how many heads and this is how many legs. So the kids learn the method, whereas the parents don't know it, so the parents are drawing out all the little horses and ducks. I think-but that's a manipulative-even though it's writing it, that to me is a manipulative. It was very time consuming. It would be a lot easier if you just tell them the formula and the rule, and just do it. But if they don't get why they're doing it, it's no good. (SD, Interview 2, p. 8-9)

Sue asserted that students must understand why they are doing a certain procedure but made this claim in the context of explaining that using the procedure is a better way to solve the problem than a self-generated method.

Another example of this tension between wanting students to understand and wanting students to calculate comes from Sue's description, from her second interview, of why quality math lessons ask students to work on only one skill at a time:

> I would definitely still stay with one skill, because I believe that mixing the skills gets kids more confused. And I definitely would have manipulative time to work through it. And then I would have small practices, because I think it's really good to be able to use tiny numbers where you actually know the answer. So as you're doing it you're getting self-gratification that you're getting it right. But then have numbers there's really no way they could figure it out. And see if now they can do it on their own. (SD, Interview 2, p. 9)

Sue believed it was important to give students difficult numbers in a problem, after working on the skill with manipulatives, to ensure that they could "do it on their own" (SD, Interview 2, p. 9). By this, Sue really meant that they could use the procedure rather than the manipulative or their conceptual understanding to solve the problem. There is nothing wrong with this idea that students must master the algorithm. This example does highlight, however, Sue's belief that until one can use the algorithm they cannot do it on their own. "Doing it on their own," to Sue, is a procedural task.

Sue's discussion of manipulatives in her second interview showed a change in her beliefs. Sue began the study thinking that manipulatives were an important part of instruction in the lower elementary grades, but not after kindergarten or first grade. Her experience using manipulatives in the professional development, particularly subtracting a negative number using manipulatives, helped Sue see the value of manipulative work for building conceptual understanding. It is unclear how often Sue implements this belief in her instruction.

One instance in Sue's second interview highlights her shift in thinking towards the conceptual:

> It happened last night. I had two students that are definitely struggling math students, and [the students were working on] adding and subtracting mixed fractions, and how they had to make it a proper fraction. And they're like, "No, no, no, but let's take the whole number and the whole number. We'll take those away. Then we'll deal with the fractions." I'm like, "You can't do it that way." But instead of telling them the rule, and this is how you do it, I was actually making little chunks and drawing it out. We were working in eighths of apples and the girl goes, "How can you have eighths of apples?" So I drew out 30 some eighths of apples and then I circled them and made little like stems on the apples. And I actually didn't teach the rule. I taught the concept. And I would honestly-I would have never done it before. I would have said, "No, this is how you do it." (SD, Interview 2, p. 3)

Sue described having more patience and understanding as a result of the professional development, especially when students struggle to learn the material. "I really felt I was a better teacher to those two little girls last night having had taken the class. I think I'm still a good teacher, and I've been good, but I think I was just a little bit better and a little more patient" (SD, Interview 2, p. 26).

Sue scored 1.96 standard deviations above the norm on her pre-training MKT assessment and 0.33 standard deviations above the norm on her post-training assessment. Sue's IRT scores were lower on her post-training assessment than her pre-training assessment and the difference between the scores is outside the standard error of measurement (SEM) of the test (see Figure 6). This indicates that there is greater than a 68% chance that Sue's true score on the post-test test is lower than her true score on the pre-test.

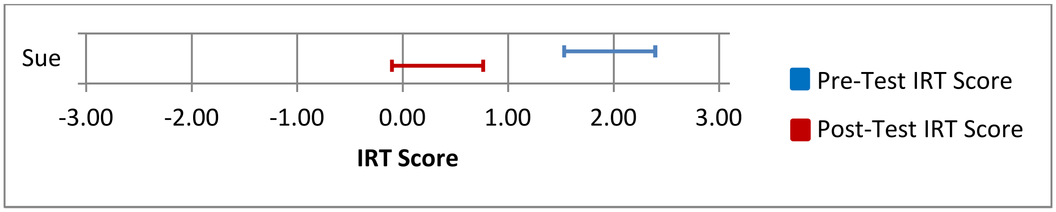

Figure 6. Sue's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

The reasons behind this change are unknown. Sue did describe her thoughts about the post-training assessment in her second interview: "I thought it was an accurate assessment, but I felt like in the first one [the pre-training assessment] it was more testing my math abilities versus the second one I felt it was really testing me as a teacher of how I would teach the same math things" (SD, Interview 2, p. 28-29). The types of questions on both forms of the assessment were similar to each other, so Sue's comment says more about her change in viewpoint than about the test itself. Sue may have been seeing the grey areas when asked about particular problem solving or teaching methods and so was over-thinking her answers. Perhaps she had a bad day. We will never know; Sue's scores do not provide any insight into the reasons behind her drop in performance.

Mary. Mary Winship is a Special Education teacher with 30 years of experience, 25 of which include mathematics teaching. At the time of the study she was teaching fifth and sixth grade mathematics, reading, and writing in a pull-out setting, and providing classroom support in the content areas. Her mathematics instruction was individualized for each student. Mary completed a teacher preparation program and is certified to teach with credentials in general special education and intellectual developmental disabilities. She majored in English and speech/drama in college and has a Master's degree in curriculum and instruction in literacy.

In her first interview, Mary discussed her belief that students need to be "taught to mastery" (MW, Interview 1, p. 5). She has found that often her students have been asked to move on to a new skill before they were ready to because the teachers were following a pacing guide. She felt very strongly that teachers should not just move on because the pacing guide says to, but should work to assess student progress and move on when students are ready. She acknowledged that this is hard to do especially with larger classes, but said it is worth doing.

Part of helping students master the material, Mary said, involves allowing students to work with concrete situations and manipulatives before moving on to abstract concepts. Mary believed that manipulatives can help students learn concepts in a concrete way. She believed that the students "get it" (MW, Interview 1, p. 4) much quicker because they can see, feel, and use their gross motor skills to experience the mathematics. Although she did say that manipulatives don't work well for every student, she firmly believed in the need for more time to be spent working at a concrete level before moving to an abstract level.

Mary monitored the progress of her students weekly in order to see if her students had mastered a particular skill, and if they had maintained other skills, before moving on. Her progress monitoring assessed concepts and applications, as well as computation. Mary said she goes over these scores with students and looks for patterns. She said it is important for teachers to monitor student progress and for students to be partners in this process.

Mary worked with students who have had a very low level of success with mathematics. Mary believed in making students “part of the team” and allowed them to discuss the process and progress of their mathematics. She believed that when students feel they are part of a team they are more invested in their learning. This team approach helped Mary confer with students about their areas of strength and weakness, rather than critiquing students for their mistakes. Mary believed it is important that students can trust that she will not “nail them for not getting something right away” (MW, Interview 1, p. 10), but that the students know that they will work together to make sure that they understand it. “No one wants to feel like a failure” (MW, Interview 1, p. 11) Mary said in her first interview. Mary wanted the students to be internally motivated and accomplished this through discussions with the student to analyze their work and progress together. She believed that working with students as a team helps students build their confidence. “I hope they aren’t so afraid of math like I am,” she says, “I want them to think that ‘Yeah, I can get this. Yeah, this is possible’” (MW, Interview 1, p. 15).

Mary began the study very interested in improving her level of mathematics knowledge. In her first interview she said:

> The only way I can get to be a better teacher of math is to increase my own education, the quality of my own knowledge of math. So if I am more comfortable doing a certain process I am going to be able to translate that to working with kids and be able to explain it. (MW, Interview 1, p. 15)

Mary talked about her “knowledge of math” as synonymous with “doing a certain process” (MW, Interview 1, p.15). Although Mary discussed using manipulatives and concrete tools in her instruction, those manipulative and tools were used to reinforce procedures. This highlights Mary’s more traditional focus in her mathematics instruction. The theme running throughout Mary's pre-training data involves working together with students to build mastery of procedures.

Mary scored 0.66 standard deviations above the norm on her pre-training MKT assessment and 0.33 standard deviations below the norm on her post-training assessment. Mary's IRT scores were lower on her post-training assessment than her pre-training assessment and the difference between the scores is outside the standard error of measurement (SEM) of the test (see Figure 7). This indicates that there is greater than a 68% chance that Mary's true score on the post-test test is lower than her true score on the pre-test.

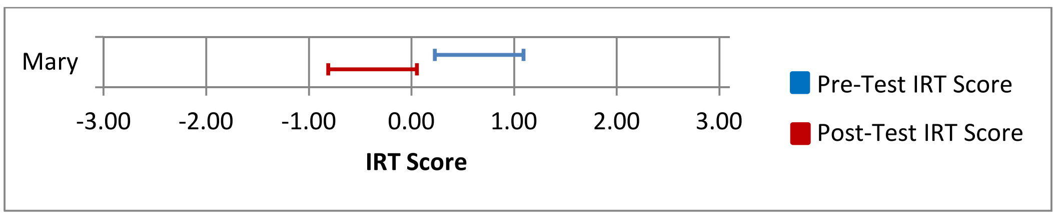

Figure 7. Mary's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

Mary experienced a technical problem when taking the post-test using the automated TKAS system. The system allowed her to answer only four questions on the post-test before it malfunctioned and ended her test. Mary alerted me to the situation and I was able to have her re-start the test. Mary was flustered. There is no way to tell if this experience impacted Mary's ability to focus on the test or if her scores would have been different had the first administration of the post-test gone smoothly.

Mary did feel as though she gained content knowledge through participation in the professional development. In her final journal entry, Mary wrote:

> This math window continues to squeak open slowly but surely in terms of my understanding and application of concepts. This is an entirely different way of teaching and understanding math for me and I still feel like those more predisposed to a "math mindset" still have the advantage, but I am starting to see that maybe I could have a "math mindset" also along with the other students who hate math.
>
> Since I have begun this course of exploration, I am better able to model math concepts in a variety of ways using arrays, tables, pictures, etc. with more flexibility and am beginning to see how language and math are related. What I mean to say is that prior to this I tended to put math in a separate area from that of language/writing/prose/etc. Now, I am not so sure where the delineation lies. They are not as separate and distinct as I once thought. They may be more similar than dissimilar. As we discuss conceptual issues and questions in books and in writing, so we can in math also. The only thing is that you will end up with the same conclusion (answer) because someone has already figured that one out. The ways you get there are also fairly defined. (MW, Journal 3, p. 1)

Despite learning more mathematics, Mary still felt the she had a journey ahead of her, as indicated in her second interview:

> I am not confident in my explanations thereof. I might be able to do it, but can I explain it? And if I can't explain it, to me that means I haven't really mastered the process. I mean, it all goes back to the teacher education part, and how did it... like with staff development and professional development and how do you boost that? How do you support and educate at the same time? (MW, Interview 2, p. 15)

This uncertainty about the explanations behind the procedures caused Mary to be less confident in her "own abilities as a math teacher" (MW, Interview 2, p. 7). Mary realized how much more there is that she did not know. She expressed frustration at her own lack of knowledge and the lack of opportunities for her to gain the knowledge that she was missing. "It's just something I've really been thinking about," she said,

> and it makes me sad, because I don't want them [students], as I think I said in my other interview, I don't want them to feel about math the way I feel about

> math. But then am I making them feel about math the way I feel about math because of my lack of knowledge? I hope not. (MW, Interview 2, p. 11)

Mary said that she did learn math in the professional development, specifically,

> I feel I learned more options in the way to approach a problem, of clarifying. I feel like I learned the whys that something works. Now, some of the whys went over my head. It's like "Oh, okay." Some of them, I know them, but I'm not quite sure I could explain them. So there are several different levels of understanding the whys. But I think as an educator, that's what is important, to understand the whys. (MW, Interview 2, p. 17-18)

After reflecting upon this and her concerns about her ability she added, "And maybe it's not so much a doubt in my own ability, it's a doubt, can I explain the whys or the rationale behind? Perhaps that's a more explicit feeling" (MW, Interview 2, p. 18).

Mary's lack of confidence in her abilities appeared to shape her view of a quality math lesson. "A quality math lesson," Mary said in her second interview,

> is building that confidence in the students. Even if they're wrong, they're still confident in the process. A quality math lesson is going to encourage them to persevere, encourage them to question the teacher as to the whys.… A quality math lesson, you can do something and you go away and you say, "I still don't get it." But a quality math lesson will, again, come back and address that until you do get it. (MW, Interview 2, p. 30)

Mary did not want students to leave her classroom feeling the same way about math that she did.

Mary's beliefs about quality mathematics lessons were not only shaped by her own experience with mathematics as a child, but also as an adult student in the professional development training. The professional development served to highlight beliefs that Mary already said she had rather than creating new beliefs. In her second interview, when asked if her definition of quality has changed over the course of the professional development, Mary said,

> No. If anything, I think it's deepened in the fact of how much more I have to... I've always known that quality needs to be individualized, it needs to be student-driven as well as teacher-driven. But I guess I just find that it needs to incorporate more of those higher-level skills than I perhaps had considered before. I kind of knew it, but I didn't really. (MW, Interview 2, p. 32-33)

In her third journal, Mary wrote:

> In the classroom, I am looking at my students beyond their ability to compute and calculate and am focusing on their ability to incorporate the whys and wherefores in order to increase overall understanding and ability to apply the understanding. I am able to do that because I am starting to see the whys and wherefores behind what is going on in a specific problem. I need to understand the processes well enough in order to guide my students as they discover their own understandings. (MW, Journal 3, p. 1)

Mary expressed the need for students to discover their own understandings, but felt unable to do this without mastering the understandings herself.

Perhaps this statement, from Mary's second journal entry best sums up the changes in her beliefs about quality lessons: "my attention has been drawn to the necessity of checking in with students and honoring the thought process involved" (MW, Journal 2, p. 1). Mary began the study discussing the team approach to building students' math knowledge, primarily of procedures. She began to develop a personal understanding of the "whys and wherefores" (MW, Journal 3, p. 1) behind these procedures. Mary did not gain enough confidence in her own abilities in these areas to make major changes to her instruction, or even to her beliefs about instruction; but she did change her level of awareness. This is summed up in her words about "honoring the thought process involved" (MW, Journal 2, p. 1). Mary has taken a step in the conceptual direction. It is small, and it is tenuous, but she has taken it nonetheless.

Bethan. Bethan O'Connell taught sixth, seventh, and eighth grade mathematics and social studies to students who have been identified as "at risk" (BO, Interview 1, p. 1). At the time of the study she had 28 years of teaching experience, all at the middle school level, and had taught mathematics for exactly half of her career. In college, Bethan majored in special education and minored in sociology. She also holds a Master's degree in special education. Bethan completed a teacher preparation program before entering the profession and is certified to teach as a generalist at the middle school level and as a teacher in special programs, such as those with at risk students. Teaching is Bethan's first career.

Bethan believes "strongly that the students must interact with the materials used to teach a new skill" (BO, Lesson Plan 1, p. 12). She accomplishes this through the use of whole body movement, manipulatives, pencil/paper, and the computers. In the reflection on her first submitted lesson plan, Bethan wrote,

> I chose this lesson because it involves several layers of learning: whole body, tiles, pencil paper, technology. With the diversity of learning styles found in one classroom, it is important to allow students to experience from the concrete to the abstract. … By progressing from the whole body to the pencil/paper to the technology I feel that all [students] will have an opportunity to be successful and build confidence with this skill. (BO, Lesson Plan 1, p. 12)

This idea of whole body learning permeated Bethan's submissions and interviews. Bethan called this a "multimodal approach" (BO, Interview 1, p. 6).

Bethan often discussed using this multimodal approach to develop a progression of learning from concrete ideas to abstract ones. When using concrete tools, Bethan said in her first interview, the students “can see it in front of them visually. They kind of build it in themselves and feel the security and at what level they are ready to let go of the concrete – and some of them don’t” (BO, Interview 1, p. 2). She described allowing different students to solve addition of signed number problems in different ways and with different tools depending on their readiness to move from the concrete (i.e. the number line or tiles) to the abstract (i.e. paper/pencil). She said she strives to move students from the concrete to the abstract “all the time, if I can” (BO, Interview 1, p. 2).

In her first interview Bethan discussed developing her lessons “to build in success at every level as you scaffold so that by the time [students] get to the pencil/paper, they feel pretty confident that they are being asked to do something that they’ve found success [with] on another level” (BO, Interview 1, p. 1). Bethan felt this success was important because taking risks is a part of learning. “When the students are at risk, they don’t want to take a risk” (BO, Interview 1, p. 4), she said. Bethan described intentionally working with her students to learn to take risks as a part of learning. By this, Bethan meant that she worked with students to attempt and participate in the solving of mathematics problems. Her students have typically not completed their homework in the past, so Bethan assigned homework that she was confident her students could do in order to provide a positive experience with it. She asked students to do their homework in a math notebook. This provided opportunities for Bethan to show her students, by looking back in their notebooks, “how successful they’ve been all the way through” (BO, Interview 1, p. 7).

This idea of building student's confidence through success was a recurring theme for Bethan and correlated with her expression of student mastery of the material. Rather than talking about mastery, Bethan discussed the idea of students "owning" the material. The phrase "own it" (BO, Interview 1, p. 4-6, 8) reappeared again and again throughout the first interview. Bethan discussed constantly reassessing students because they may do well at one point but may not later on, an indicator that her students don't yet "own the skill" (BO, Interview 1, p. 4). She added that quality lessons assess students differently so that students have a way to show what they know. "Not assessing more, just assessing differently" (BO, Interview 1, p. 4). She used an example of students who are able to show the teacher the process but are unable to write it down. "Show me, … get it out somehow so that I know in your way of processing," she said, "you own it" (BO, Interview 1, p. 4).

The theme running throughout all of Bethan's pre-training statements is a multimodal approach to teaching and learning. When asked, in the first interview, what the phrase "quality mathematics lesson" meant to her, Bethan quickly replied:

> Multimodality approach, that's what it means to me. At the level that they will be successful. And know your students. You can't know quality unless you know what you are aiming the quality towards, because one person's quality, may not be the other's. (BO, Interview 1, p. 9)

Bethan's level of content knowledge is relatively typical for elementary math teachers. She scored -0.68 on her pre-training MKT assessment which means that she scored 0.68 standard deviations below the norm. She scored 0.33 standard deviations above the norm on her post-training assessment. Bethan's IRT scores were outside the standard error of measurement (SEM) of the test (see Figure 8) meaning that there is greater than a 68% chance that her true post-assessment score would be higher than her true pre-assessment score. This difference, however, is slight.

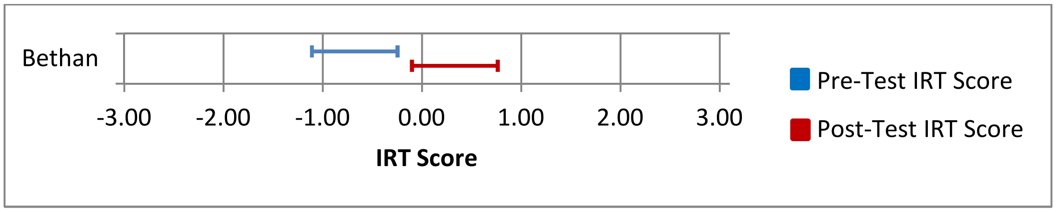

Figure 8. Bethan's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

Bethan expressed gaining content knowledge during the professional development. In her second interview she specifically mentioned gaining an understanding of multiplication of fractions as a "part of a part" (BO, Interview 2, p. 1) and the larger idea of "decomposing" (BO, Interview 2, p. 5) mathematical ideas and procedures. By decomposing, Bethan meant that she learned how to break down skills into their subcomponents and how to reassemble those pieces into the knowledge as a whole. She used the example of finding surface area. Bethan said that she used to expect students to understand how the steps of finding surface area were "all under one umbrella" (BO, Interview 2, p. 5), but that she has learned how to make that connection explicit for her students and to "break things down a lot more than I did prior to the course" (BO, Interview 2, p. 5).

Bethan described the pattern of her lessons before the professional development as: give the procedure, give the steps, do the problems. After the professional development, Bethan has changed her lesson structure to have the class talk about the concept first, then she provided examples, and then she asked her students to find or provide examples, discuss the examples, and present their examples. Only after this student exploration did Bethan go back and give a structured example. “The course,” she said in her second interview “in that way has helped me as a classroom teacher; to help them [the students] to explore it themselves instead of just sitting back and waiting for somebody to do it” (BO, Interview 2, p. 12). Bethan said that when the students are able to understand the topic through their own work before seeing the structured example, “then they don’t need my structure. I facilitate, don’t instruct” (BO, Interview 2, p. 12).

Bethan’s new structure allows for students to share their thinking and processing in addition to Bethan showing students how to solve problems. Her shift from “instructing” to “facilitating” means that students are sharing in the development of the lesson and in the ways in which information is presented. This shift in the locus of control of the lesson represents a significant change in the structure of Bethan’s lessons. Despite this change in structure, Bethan’s change in pedagogy appears tenuous. This became clear during her second interview when Bethan said:

> I think [my beliefs about what makes a quality math lesson] were more reinforced and added to. With the group that I work with it’s not like a regular classroom so I guess my orientation is in one direction as far as visibility. Knowing what I deal with and what will work and what won’t work and, not that I don’t try different things, of course I do, but I know that you have to start with the whole body involvement and work your way on down and with the type of child I work with, it’s just how it works. If it works and it’s not broken then don’t fix it. If they’re feeling good about themselves and they feel they can do the math and they can own it and they can turn around and help somebody else with it then go for it. (BO, Interview 2, p. 2)

The biggest change Bethan experienced was that of perspective. Bethan wrote in her second journal entry that her beliefs about quality lessons hadn't changed per se, but had been reinforced and added to.

> Understanding Elementary Mathematics has reinforced many of the ideas I initially stated. It has added things to my ideas of what a quality math lesson is. I have learned that students look at math in many different ways to arrive at the same answer. There is not just one explanation to a presentation of a new math concept. I have learned that as teachers we must be very tenacious in our observations of how students think and in our assessment of their work. (BO, Journal 2, p. 1)

Before the professional development, Bethan thought of the multimodal approach as a way of teaching that incorporated many of the ways students learn. Within the structure of the professional development she began also to think about the multiple ways in which students think. Although subtle, this shift is important because it allowed Bethan to recognize that she must change the ways in which she listened to students, not just in the ways she instructed students.

The beginning of this shift in focus from student learning to student thinking was first evident in Bethan's second journal. She wrote:

> Being involved in the Understanding Elementary Mathematics course has certainly enabled me to look at math differently. It has created new ways of thinking in regard to how students look at math problems. It has made me think more about how I present math concepts and wonder if I am reaching all of the students. What more do I need to do to enable all students to feel comfortable enough and confident enough to enter more actively into the discussion and interaction of the classroom? (BO, Journal 2, p. 1)

Bethan was beginning to notice that there was a connection between what she was doing with her instruction and how students were interacting with the mathematical material and bringing their thinking to the lesson.

This shift in her understanding of student thinking was accompanied by a shift in her understanding of her own learning. In her third journal, Bethan wrote:

> I have gained knowledge because I have learned to look at math in different ways that I have not looked at math before. I was teaching math the way the text/curriculum coordinators presented it. I think that I did not understand many of the ways things were broken down or decomposed. I would not have expected the students to. I have realized that math must be looked at in many different ways, (for better understanding and to reach all students) but this class taught me to look even further. This class has taught me to ask myself more questions and listen and watch how students process and demonstrate their knowledge of mathematics. (BO, Journal 3, p. 1)

This connection between her own knowledge and her ability to examine student thinking dovetails with Bethan's newfound control over her teaching methods. Bethan did not say, in her pre-training data, that she was teaching the way others told her to teach, nor do I think she would have recognized that fact had I asked her about it before the training. By pointing it out in her post-training data, Bethan called attention to the fact that she now felt that she could decide, based on her students' thinking and her own understanding of mathematics, to change the methods used to teach her students. This feeling, though, appears tenuous.

In her second interview, Bethan talked about the ways in which the professional development provided a new learning experience for her. This new experience, she said, gave her more ways of thinking about teaching mathematics. At the same time, she recognized the impact that her previous learning experiences have had on her teaching.

> You assume that you were taught one way and you did it that way and you never really experience and look for any other way because it was working. … the course certainly was [different from how I was taught mathematics] because in my stage I mean I got my Master's [degree] years and years ago and we were

> taught certain ways. You learn to change your way of approaching but a lot of times if you want it to work you fall back to the way you were taught yourself. … It's like when you raise children you never think you will say what came out of your parent's mouth. And then you realize one day "Oh my god, that's what my mother said!" (BO, Interview 2, p. 5-6)

The tension between Bethan's statements about changes to her teaching and statements about the ways in which she falls back to the ways in which she was taught indicate that perhaps Bethan' beliefs and classroom practices are in flux and that it is still unclear whether or not the changes Bethan indicated will take hold over the long term.

Ireane. Ireane Croteau taught special education in grades five through eight. She provided students with supplemental instruction in mathematics, language arts, and science. These students were in a pull-out setting for mathematics which is taught by another mathematics teacher. Ireane worked with the students during their academic support class daily in order to focus on the mathematics skills that the students' primary mathematics teacher identified. At the time of the study, Ireane had been providing special education services at the middle school level for five years and before that spent ten years teaching life skills to high school students with more severe disabilities. Ireane majored in education in college and minored in speech communication. She completed a teacher preparation program as part of her undergraduate degree. She does not hold any graduate degrees. Ireane holds general special education teaching credentials in addition to a specific credential for working with students with intellectual developmental disabilities. Teaching is her first career.

In her first interview, Ireane talked about her poor experiences with mathematics. As a student, Ireane found that word problems never made sense, but that if she drew out the pieces of the problem, it helped her understand it. She said she sometimes did not know what to do with the parts once she had them, but she found that drawing it out “took away the fear of the problem itself” (IC, Interview 1, p. 7). She did not particularly like math growing up because she “wasn’t successful” (IC, Interview 1, p. 7), but did like teachers who were “organized, structured, no fooling around, said what they wanted, gave you a study guide, gave you a test on what you were asked to do” (IC, Interview 1, p. 7). Ireane shared that she had a very low confidence level in her mathematics ability and had signed up for the study so that she could learn mathematics content in order to better help her students. She had participated in a different professional development program in mathematics content the summer before and had found it too difficult for her. She said she signed up for the study because she needed help learning the content so that she can teach it.

Ireane’s own assessment of her low level of content knowledge was confirmed by the MKT testing. She scored 1.52 standard deviations below the norm on her pre-training MKT assessment and 1.39 standard deviations below the norm on her post-training assessment. Ireane’s scores on these tests are essentially the same (see Figure 9); the difference is not meaningful. These scores mean that Ireane’s level of content knowledge was lower than many elementary teachers. This may have made it especially difficult for her to work with the upper elementary content that she encountered in her teaching position.

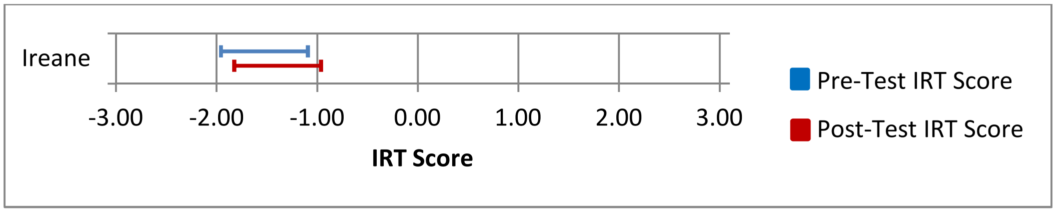

Figure 9. Ireane's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

Ireane discussed, in her first interview, how she spent most of her instructional time working with students to fill gaps in their functional skills such as telling time or counting money that are required for life. These skills are not necessarily part of the student's daily math class. "So often my lessons are taught and re-taught and re-taught that it's sort of like a continuum. I don't feel like my lessons are finished" (IC, Interview 2, p. 12), Ireane said. "You always have to take it to the next level. Sometimes you just want to say, 'okay, I'm done that' and check it off. But math doesn't seem to have that check off point, does it?" (IC, Interview 2, p. 12).

When teaching, Ireane said it is important to break down skills into small chunks focusing on one skill at a time, and then re-combine these to make up the larger skill. Review every day is also important to Ireane.

> I think having the repeat and review every day is very, very, very, very, *very* [important]; I forgot to mention that. I think repeating – even if you already know that they know it, there's someone who doesn't; and the more you repeat it every day, repeating the same thing every day for two minutes will really make a difference. (IC, Interview 2, p. 8-9)

Ireane said it is important to use the child's learning style to decide whether to use manipulatives, paper and pencil worksheets, or the whiteboard. Ireane describes this as "whatever their learning style is, I just sort of follow their lead" (IC, Interview

2, p. 5). Group work can sometimes be powerful, she said, “but I also think that’s a learning style, too” (IC, Interview 2, p. 8). Many of Ireane’s students can do rote computation, but cannot apply the skills to real life. Ireane wants to make “those applications make sense” (IC, Interview 2, p. 14). Ireane said she often relates the mathematical ideas she is teaching to something the students are familiar with such as money. “Everything can relate to either money or food. It really makes it relevant, meaningful, motivational, and the kids respond well when you’re talking in a language that means something to them” (IC, Interview 1, p. 1)

Ireane tries to make the math sound light and easy and fun for her students. “Learning can be very hard,” she says, “and our poor kids, learning can be very, very hard for them and yet they don’t give up. They are so happy-go-lucky, still counting money in eighth grade but they just don’t give up” (IC, Interview 1, p. 12). At the end of the lesson she wants her students to feel as if it’s not as intimidating and feel like they are getting it, that they leave feeling better about it.

> It’s almost like an infomercial. I often feel like I’m setting up a lesson and I’m like, “Okay, look, it’s only just *two* steps,” and trying to sell them on “look how cool this is, it’s really easy.” But it has to be easy for me otherwise I can’t do it. It’s select things I can do that to. (IC, Interview 1, p.10)

Before the professional development, Ireane thought of mathematics as a series of procedures to be memorized and carried out. She likened these procedures to recipes:

> I think once I know the content, I can look at Math in Focus [the textbook series her school uses] and say, “okay, show me this another way.” But I don’t even have one way, so I just want to start with one way and master it. And I think once that’s mastered, then I think you are more receptive to saying, “well, okay, let me see another way.”… It’s kind of like baking. Everyone has their own recipe, their own way of doing it. So give me one that I can start with and be

> successful on my own and then, "okay, I want to experiment now. Let me see what you do and how you do yours." It's kind of like that. (IC, Interview 1, p. 16)

Ireane believed that if she were given one way to solve certain classes of problems she could learn the mathematics. She thought of her students, who also struggled with math, as like her and thought that they also needed their lessons to be broken down into steps where the students are given one way to solve a problem which they work to master. By the end of the professional development, Ireane had completely changed her mind.

Ireane's learning experience in UEM had a huge impact on her. In her first journal, Ireane was still feeling insecure in her abilities:

> When given word problems to solve my very first reaction/response is to freeze up. Because I lack prior knowledge in basic problem solving/thinking skills and strategies, my ability to respond to prompts is hampered. Participating in UEM will hopefully address weaknesses so learning can continue from a stronger base. (IC, Journal 1, p. 1-2)

By her third journal, Ireane was recognizing how much she had learned:

> Do you feel you gained mathematical knowledge, confidence, and/or new ways of thinking?
>
> If you had asked me that question prior to taking today's posttest I would have had a hard time answering that question. Knowing I still do not have solid algorithms for solving problems related to much of the 7th and 8th curriculum makes it difficult for me to feel knowledgeable and confident. Throughout this course I could see how little number sense I have in comparison to my peers. But because of the safe and encouraging learning environment [the instructor] created none of that mattered so I came to class with hopes of learning but not really expecting to.
>
> During our exiting posttest I found my knee jerk response to turn away and run from word problems did not kick in. Why? I wondered. As I went through each question I was puzzled as to why they were not asking me to solve the problems… I wanted to solve them! For the first time I actually understood what question was being asked!

> It is an empowering feeling to begin reading word problems for the first time with even the smallest amount of confidence. Taking the posttest I experienced what it is like to understand the question being asked. I could see what was taking place and had strategies to solving them. Evidence of new ways of thinking had truly been gained. I used to think simply knowing algorithms would help me understand math. It was my experience in Understanding Elementary Mathematics that taught me how seeing mathematics is truly the way to learn, understand, and ultimately think. (IC, Journal 3, p. 1)

I asked Ireane about this during her second interview:

> Q: I read through your journal and thought it was awesome. So, the first thing that jumped into my head is, "Okay, Ireane, you know me pretty well. Are you just saying this because it's what I want to hear?" [Laugh]
> IREANE: No. In fact, when I wrote this word knee jerk, I remember the very first day when we walked in [to UEM] and there was a word problem. I wanted to get up and leave and said, "I don't want to be doing this." Because they're just so intimidating.
> Q: So, tell me, you talk about this big difference for you.
> IREANE: The biggest problem for word problems for me was that I didn't understand the question or at least I understood the question incorrectly. So I never was close to solving it and I was never confident with my strategies. But, through the course, and I don't know how it happened because I couldn't tell you which... there wasn't an a ah-ha moment, but somehow the questioning, the thinking, the experiences let me understand what was being asked at a conceptual level. I could see it and then you know how to solve it, you understand the question, then you just have an answer, you know how to figure it out. Even if it's wrong, you know what. So, I just thought that was huge for me that I could experience that. (IC, Interview 2, p. 1-2)

Ireane's third lesson plan was on teaching word problems, a concept that she was willing to tackle because she was "not scared of them anymore" (IC, Interview 2, p. 4), at least ones that are not "above [her] competency level" (IC, Interview 2, p. 4). Ireane's learning transferred into her teaching. She said she chose the lesson on word problems because she would like to:

> do more word problems with my kids and hear how everyone comes up with any answer and they will be learning without having a formula in place. They're still getting something from other people's experiences. And maybe communicating their understanding is a way of deepening it as well, having to share it with somebody else. And that was a big part of [UEM] that everything had to be communicated. It wasn't okay to say that's the right answer. How'd you get it? Now get it a different way. (IC, Interview 2, p. 4)

This experience, of understanding word problems and gaining conceptual understanding had a dramatic impact on Ireane's beliefs about teaching and learning. In the second interview I asked Ireane about her change in belief about the value of multiple solution strategies:

> Q: I remember in your first interview you talked about: "give me one way; I'm not ready for more ways."
> IREANE: Yes, yes.
> Q: And it seems like you might think a little differently now.
> IREANE: It's changed. Absolutely. Because I thought just knowing the one way was going to make it so that I could learn it. But that's not the case, so I'm getting what the new philosophy in education is all about. And that it's not about getting the answer. It's not about having that formula memorized. It's about being able to transfer that formula to different settings and communicate it differently and that's something that was taught [in the professional development] and it wasn't taught by prescription, it was inferred or I don't know what the word is, but modeled. And then, as you experience what's being modeled, you get it. It's like yeah, I understand why I need to know more than one way because I don't really understand it when I just know one way and when I just get the answer. It doesn't last. So, yeah, it's interesting because I would "Yes, yeah, I hear that that's the philosophy, good. Now how do you do it?" I would just yes it, but now I can't just yes it, I get it. I don't necessarily like it, because I want to move on, but I get why I can't. And I get why our kids can't, they really need to understand it. (IC, Interview 2, p. 5-6)

Further, Ireane expressly credits this learning to the way in which the professional development was taught. By experimenting and gaining conceptual understanding herself, she learned that it was important for students to have the same experience.

Before the professional development, Ireane said in her second interview, she was "more traditional" (IC, Interview 2, p. 16). She thought quality lessons were more "teacher-directed rather than investigative" (IC, Interview 2, p. 16). She thought quality lesson plans listed the materials and gave the steps for what to do. After her experience in UEM she felt that the students "are being led to the water so to speak, rather than just saying do this first then we're going to do this, then we're going to do that." (IC, Interview 2, p. 16). She described a change in focus from teacher-directed to examining the process of student learning. "The way we created our lessons [years ago when she received her teacher training] probably are outdated, but I kind of still fall on that structure. But I see now that really it's the process behind the learning," she says, "the opportunity to process it and explore and experiment, and the safety involved with that" (IC, Interview 2, p. 17).

This change in Ireane's mathematical understanding can be seen in her description of changes she made to her first submitted lesson plan, which she modified for continued use with students. The lesson asked the students to decide which of two numbers is greater. Ireane used a place value chart and, in the original lesson plan called for the use of dimes and pennies to represent tenths and hundredths. Ireane changed the lesson plan to add rice to represent thousandths on her place value chart.

> There's a lot more of them [pieces of rice] and they're a lot smaller, and the further it gets away [from zero, to the right] it's just going, it goes from small, smaller, smaller and then just gets to the infinite. So that's why I've got the rice to show that it's decreasing…. I think with that visual, you can see the direction better.… I think that's what a conceptual understanding of math is, is you see the movement. You see, "Oh the numbers are increasing or they're decreasing, or it's about this much of them is gone," and then they just see it better. (IC, Interview 2, p. 22-23)

This idea of conceptual understanding resonated deeply with Ireane. In the second interview Ireane expressed feeling like she needed to continue to build her own knowledge so that she could make more changes to her teaching and better help her students.

> I understand the need for the conceptualization. And now it's almost like learning how to ride a bike, you need to get the feel of what balance feels like and then you can move forward…. So I feel like I've experienced balance by having the opportunity to conceptualize. But now I still know that there's some basics that I don't have, but I want to be able to take to the conceptualized level for kids. I still think I need to go one to one and just get some basics. Especially when you get to higher math, you have to have that conceptualization. (IC, Interview 2, p. 34)

Ireane's case clearly shows the impact that experiencing understanding can have on a teacher. Ireane learned conceptual mathematics through her experience in the professional development training. This experience was completely new to her and had a profound impact on her as a person and as a teacher. Ireane learned that she could learn mathematics, and through that experience came to believe that her students could also. Ireane's transformation was from wanting to break down procedures and be taught one way to believing that conceptual understanding is not only possible, but important.

Ellyn. Ellyn Dustin taught sixth and eighth grade mathematics and reading to students in a pull-out setting removed from the general education setting. Two of her classes were fully individualized, and two paralleled the general education curriculum. At the time of the study Ellyn had been teaching for eleven years, ten of which were in grades five through eight, and had taught math for four of those years. Ellyn majored in child/family studies in college and minored in psychology and Spanish. Before becoming a teacher, Ellyn worked in Social Work. Ellyn holds a Master's degree in special education during which she completed a teacher preparation program. Ellyn is certified to teach with a credential in general special education.

Ellyn scored 0.69 standard deviations above the norm on her pre-training MKT assessment and 1.37 standard deviations above the norm on her post-training assessment, indicating a higher than average amount of content knowledge. Although Ellyn's IRT scores were higher on her post-training assessment than her pre-training assessment, when considered using the standard error of measurement (SEM) of the test, the error bars overlap (see Figure 10). This indicates that Ellyn's true scores on the test are within the margin of error of the test and the difference in scores should not be considered meaningful.

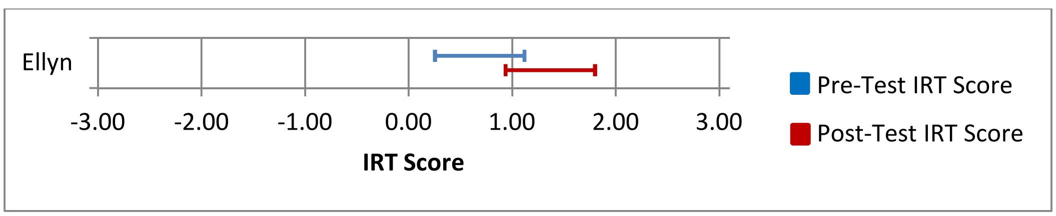

Figure 10. Ellyn's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

The theme in Ellyn's pre-training data is that Ellyn believed that, when possible, lessons should be tailored to meet individual student needs. This student focus, she noted may not be possible in "larger, general education classes" (ED, Pre-Beliefs Verification, p. 2-3), but she did feel that this individualization is an important component of quality in her small, special education classes, especially those with teacher:student ratios of 1:1 or 1:2.

Ellyn taught two classes that parallel the general education curriculum in which she tried to focus on the needs of the students in the class. In her first interview, Ellyn discussed the ways in which she modified the curriculum materials for use with her students. Although she used some of the same materials as the general education teacher, but she preferred classroom materials that: are less visually overwhelming; leave space on the page; don't require students to copy problems onto another sheet of paper; and that provide repetition of same type of problem on the same page rather than many different types of problems mixed together. She found materials with these characteristics less overwhelming for her students to use and that students' anxiety was lower.

Ellyn wanted to make sure that students in these classes did not fall behind the pace of the general education curriculum and taught the same concepts as the general education classes without being too "overwhelming" (ED, Interview 1, p. 2). Many of her students in these classes were placed with Ellyn because of anxiety and emotional issues, not because they were unable to complete the mathematical tasks. Ellyn discussed trying to provide opportunities for these students to move around and use their bodies while learning. Sometimes she accomplished this by having the students use the whiteboard for their work rather than working at their seats on their work papers. Ellyn said she likes to use manipulatives, technology, and computer games to engage these students and to help them understand the concepts. When teaching, Ellyn said, she does not want to lose sight of "making sure that kids are properly challenged and scaffolding them [the lessons] well. Those two things go hand in hand" (ED, Interview 1, p. 11).

In her first interview, Ellyn shared her belief that it is important for students to both understand the mathematics concepts and to be able to do the calculations. She shared one example:

> This one particular girl still likes to see, like if she is adding $3\frac{1}{4} + 2\frac{2}{4}$, she wants to draw it out. Sometimes I'll let her do that, so I'm trying to help her see that yes you can visualize, but sometimes you also need to do the calculations and this is the way to do the calculations. (ED, Interview 1, p. 4)

Ellyn's goals for her more severely disabled students centered on the students being able to apply math to their lives. When asked for an example of what she meant by this Ellyn said she wanted her students to encounter a situation and knowing whether to add or subtract to solve the problem and to know what those operations mean. These two examples, taken together, indicate that Ellyn, although she expressed a belief about the value of both understanding and computation, may have valued computation more than conceptual understanding.

When planning her lessons, Ellyn said in her first interview, she tries to choose materials and activities that teach a mathematical concept and to have an activity that is fun. She said one way to do this is to make the mathematics lessons applicable to life. For example, when creating a measurement lesson for one student who is a skier, Ellyn used the actual snow totals from two different ski areas where the student regularly skis. This made the activity more meaningful to the student and is applicable to the student's life. Ellyn said that lessons should be "child-friendly and child specific" (ED, Interview 1, p. 9) and that "learning should be enjoyable so that, no matter what the subject … it needs to be taught in a way where the student feels engaged" (ED, Interview 1, p. 8).

Ellyn very quickly saw changes in her own beliefs. She expressed becoming more focused on student thinking. Ellyn made changes to her lesson structure to adjust for this changing focus. Before the professional development, Ellyn's lesson structure used content from the Common Core State Standards (CCSS) (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) as the "spine" (ED, Interview 1, p. 5) of the lesson to "teach the student something they didn't know in combination with something that they did know so that they have a level of success" (ED, Interview 1, p. 6). Then the lesson should "engage [students] in some way with something that captures their attention … [to] make the lesson meaningful" (ED, Interview 1, p. 6). The lesson should have "a level of instruction for [from] the teacher [with] visuals left for the student to refer to" (ED, Interview 1, p. 6). Finally, a quality lesson provided "a time for them [the students] to try it [the mathematics] on their own" (ED, Interview 1, p. 6).

After the professional development, Ellyn's description of a quality lesson still addressed a Standard from the CCSS, but engagement and instruction portions of the lesson looked very different than they had before, and the time for student practice was more focused on application in a variety of situations, including real-world situations. The introduction to the topic of the lesson, Ellyn said in her second interview, should be hands-on and meaningful, involve exploration, and may or may not use manipulatives. This introduction should "engage their [students'] thinking" (ED, Interview 2, p. 5). The student should interact with the introductory piece and respond with "whatever they're led to want to say or think about that introduction" (ED, Interview 2, p. 5) and “students talk about what they're understanding and explain their thinking” (ED, Interview 2, p. 9).

The teacher’s role during the lesson is to:

> guide students in their exploration and in their learning and to check in to ensure that the students aren't going astray, that they aren't getting lost along the way and to bring them back and help rephrase it, revisit it so that they can get it in a way that makes sense to them. So it's not even the teacher rephrasing, it may be that they need to step back and say, ‘Well, let's get out these cubes,’ or ‘Let's get out a 3-D model,’ or something to help the student grasp it in a meaningful way. (ED, Interview 2, p. 9)

The purpose of the exploration is to have the student “gain new knowledge or confirm previously held understandings” (ED, Journal 2, p. 2). After exploration, Ellyn continued, students should apply their new knowledge in a variety of ways and in real-world situations. “Math lessons need to be connected to our lives,” she said. “We need real-life examples to work with in order to embed a concept into our mathematical consciousness. Following steps to achieve an answer to a problem is useless if we don’t know how we would ever use the steps in our own lives” (ED, Journal 3, p. 2).

Ellyn continued to talk about meaningful application of mathematics. “The most important word in all that is the meaningful part” (ED, Interview 2, p. 5), asserts Ellyn. She describes meaningful as “applying it to something in [the student’s] life that matters to them” (ED, Interview 2, p. 5). In her third journal she writes, “I think it needs to be challenging intellectually and personally, with the goal that the learner gains knowledge and confidence in their understanding of a math concept. Fun would be nice in school, but it’s not an elemental part of a math lesson” (ED, Journal 3, p. 2). This switch from fun to meaningful is one that Ellyn identified as a major shift in her thinking about mathematics lessons. “I wanted to comment,” Ellyn said towards the beginning of her second interview,

> that I remember talking with you and feeling strongly that math needed to be fun. I remember that's the first thing I said. And I don't feel that way anymore. I don't. … I feel like it needs to be enjoyable because there's – I don't want it to be a punishment to kids, but I don't feel like I need to make it into a fun class as much as it needs to be a class that feels enriching to them. And I feel that when they are enriched and they grow and understand something, that is now my definition of fun. (ED, Interview 2, p. 12)

This change in Ellyn’s focus from fun to enriching mathematically paralleled her shift in focus from procedural understandings alone toward conceptual understandings, as this exchange from her second interview illustrates:

> Q: One of the things that you talked about a couple times was how your thoughts about what make a good lesson have morphed, I guess, might be the right word. Can you tell me a bit more about that?
> ELLYN: I feel like I've changed a lot in what I seek out of a lesson from my students. Whereas before I used to be more concerned about making sure that they got to the right answer – and I wanted them [to] – I wanted to make sure I could see that they could do all the necessary steps that I had taught them.
> Q: You completely changed your mind. [Laugh]
> ELLYN: I really have too, now being more concerned with having them tell me, watching them process whatever the question is that I pose, giving them a chance to explore and explain how they get to a solution or just the processing they do to try to figure something out and I question them more deeply about that thinking. So I've grown to be less concerned about getting to the answer, [Laugh] than on finding out how students are processing and how they bring background knowledge and what their background knowledge is. And also, having them apply to more real life problems than I previously had.
> Q: Sure. Sure.
> ELLYN: That's a major change for me. (ED, Interview 2, p. 1-2)

Ellyn moved from a more traditional to a more constructivist philosophy of mathematics teaching. When asked, in her second interview, if she knew what caused this change, Ellyn responded right away "the way that the class was led and taught" (ED, Interview 2, p. 2). She elaborated:

> I haven't had a lot of math instruction, just kind of what I observed and been involved with in assisting kids as a special educator. So, because I had to learn things I wasn't sure how to do. I had to think about how I was processing it. And more instructional for me than anything, I think, was realizing how different all of us in that class did it. And that we still would all come to the same answer, if the answer was what we were seeking, but in such a variety of ways. And even all the [different computation algorithms that the instructor] put up on the wall that we had to think about. We had to go through and think about what was being done without being told what we were looking for, what was done. That was an interesting way to go through and realize that people are doing it all differently and that's perfectly fine. The goal is to find out what allows someone to come to a solution. And to figure it out if they do it in a way that's different than I do it or if they don't do it the way this step said in the book, that's fine. If they can articulate what they're doing and it makes sense. So, I've been interested to watch my students in a different way as a result of what I went through as a teacher, as a student of math. That's what made a difference. (ED, Interview 2, p. 2-3)

Although Ellyn gained confidence in her pedagogical skills for math teaching as a result of the professional development experience, she also reported that she felt less confident about being a math teacher. Before the professional development she had considered gaining Highly Qualified Teacher (HQT) status in math.[41] In her final interview, she shared questioning this route. "I need to learn how to be a better math teacher" (ED, Interview 2, p. 26), she said.

> That's one thing I've learned from this. I thought I was an okay math teacher. I thought I was okay. But now, I learned how much I don't know … and there's nothing wrong with that. There's nothing wrong with learning that. It'll give you more insight. It gives me more insight into my strengths and my weaknesses. So I just haven't figured out what I'm doing with it. (ED, Interview 2, p. 26)

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[41] Highly qualified teacher status is a designation under the No Child Left Behind law that says all teachers must have a certain level of knowledge in the content areas that they teach. A teacher who has been granted this status by their state's Board of Education is called HQT.

Later she added, “maybe I need to do more exploring if I'm going to continue to feel that I can confidently teach students, I know I need to do some more professional development classes” (ED, Interview 2, p. 29).

Ellyn recognized that she feels as though she is a better teacher in terms of pedagogy, even if she is not as strong in content as she would like to be. The following exchange from her second interview illustrates this:

> ELLYN: I feel like my teaching style has changed and that's where I feel like I'm a better teacher. So, it's a juxtaposition. I feel like I know that there's a lot I'm missing, that I would need to be a better math teacher. Yeah, I feel like I've become a better math teacher because I'm more aware of what I'm...how I'm teaching. So I've increased my confidence and decreased my confidence at the same time. I'm just realizing this as we're talking.
> Q: So, it sounds like you've increased your confidence in the pedagogy.
> ELLYN: Yeah.
> Q: And decreased your confidence in some ways in the content only because there's so much more that you feel that you don't know...
> ELLYN: Yes, Yes, right.
> Q: ...even though you know more than you did before.
> ELLYN: Yes.
> Q: That is a juxtaposition, isn't it?
> ELLYN: Yeah.
> Q: Yeah.
> ELLYN: Exactly. (ED, Interview 2, p. 28)

Initially, Ellyn was very student-centered. She described lesson quality as relating, in part, to students’ individual interests and needs. That focus on the student did not change. What did change was Ellyn’s understanding of what focus on the student should look like. Before the professional development, Ellyn’s mathematical focus was on the product, the answer at the end of a question and making sure that students could find that answer. After the professional development, Ellyn focused on the process of learning mathematics; she believed that lessons should include time for students to talk about their understanding and that teachers should guide students towards deeper understanding. In her final interview, Ellyn described with joy the difference that has occurred in her teaching: “I have seen them [my students] light up when they've just done their own self discovery and realize they knew something that they didn't know they knew. [Laugh] I never knew that before, that kind of thing” (ED, Interview 2, p. 13).

Marie. At the time of the study, Marie Gilmore taught four sixth grade math classes that were heterogeneously grouped and a test preparation class for approximately half of those same students. She had been teaching for nine years, seven of which included mathematics. Marie majored in business administration in college and entered the field of banking. Teaching is her second career. Marie completed a teacher preparation program as part of her Master’s degree in elementary education. She was certified to teach elementary school and middle school mathematics (grades 5 through 9).

Marie has a higher level of content knowledge than is typical of many elementary school teachers. Marie scored 1.43 standard deviations above the norm on her pre-training MKT assessment and 1.96 standard deviations above the norm on her post-training assessment. Although Marie’s IRT scores were higher on her post-training assessment than her pre-training assessment, when considered using the standard error of measurement (SEM) of the test, the error bars overlap (see Figure 11). This indicates that Marie’s true scores on the test are within the margin of error of the test and the difference in scores is not meaningful.

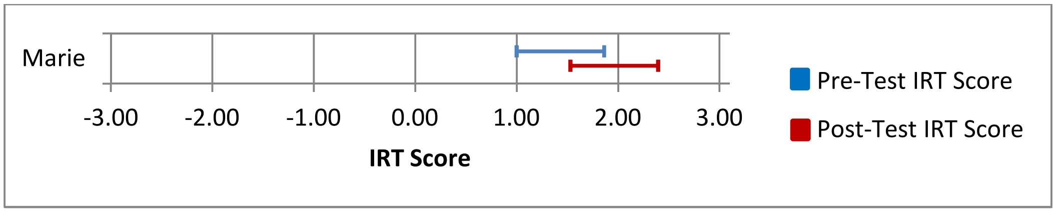

Figure 11. Marie's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

Marie loved math growing up because she loves a "puzzle" (MG, Interview 1, p. 10) and she loves "a right answer" (MG, Interview 1, p. 10). She said she is "not a stellar math person" (MG, Interview 1, p. 10), but she loves the process and the result. It wasn't until graduate school that Marie discovered that "math was not just a sum of all the things you have to do to get to the end" (MG, Interview 1, p. 10), but that math could be a "thoughtful pursuit" (MG, Interview 1, p. 10). Even then, Marie said that "what really changed my perception of what a math lesson should be is being in the pit" (MG, Interview 1, p. 11). She realized that different students can have very different needs and teaching to any one set of students "is just not going to work" (MG, Interview 1, p. 11). Good lessons, Marie said in her first interview, need to "get everybody to the table and participate. I do think you need to be participating and engaged in order to learn your best" (MG, Interview 1, p. 11). Marie said that this excitement helps the students change their mindset and not "have to drag through this" (MG, Interview 1, p. 2). Media, Marie found, makes a good hook to help students get excited.

In her first interview, Marie said that having something to visualize helps some students, but that not all students need the visual. She says there is a need for both the abstract and the concrete in a lesson.

> At least half of the kids they never need to see that [the item or manipulative], but for some of the kids, they really like to see it. Like I'll throw it in the middle of that group because I know there's three kids in that group who will, they'll count. … Where there will be kids in here who would be like, "Who would ever need to do that?" But because you have to reach every level, I feel like I try to do something – in a perfect world this happens in every lesson – I want to go real-world application because I have a whole level of kids who are ready for that, and I want to go concrete because I have a level of kids who need that. So, I'm stuck; that's my best chance at differentiating. In a 45 minute math class differentiation is like, 26 kids in 45 minutes; and it's not realistic to me. So if I can make both pieces be available in the same lesson I feel like you might not get that and you may not need this, but at least I have a little something for everybody. (MG, Interview 1, p. 4)

This idea that every kid needs something different and that Marie needs to provide something for everyone helped Marie do more than provide a procedure by which to solve problems. However, her lessons were focused at leading students to that procedure.

Marie believed that each lesson should have some time for independent work so that there is variety in what is happening in the classroom, but also so that students have an example or model for their homework. Homework, according to Marie, should only include problems where the students have an example to follow or have practiced the type of problem already. She said she does

> not want the parents to teach them math because I don't want them to learn different ways than I want and I also communicate with the 7th grade about how they would like to see them solving equations and so we want to try to keep them in this cocoon of doing things in the way that we will progressively continue to teach it. (MG, Interview 1, p. 6)

Marie "need[s] them to have time to do independent work so that they have time to ask questions" (MG, Interview 1, p. 6).

Marie believed it is important for students to "own" (MG, Interview 1, p. 6-7) the material by the end of the unit. In a beginning lesson, students need to begin to think about the topic, then work with it, then eventually own it. To this end, she has implemented graded class work where the students can use their notes, their homework, or the teacher to make sure that they are moving towards mastery of the material through their own personal engagement with the material. The test or quiz checks for that final mastery.

Marie is wary of the use of group work in mathematics because

> too many leaders do all the work in the groups. … Some collaboration [is fine]. Sometimes I feel like that makes the kids feel dumb because there's always the kid that can do it and they can't do it. … I'm very afraid of them feeling dumb." (MG, Interview 1, p. 8)

To avoid this, Marie said she gave lots of private praise when students do something well. She tried to build rapport with students so that they feel as though they can come to her if they need help. She generally did not call on students without warning. When she did, she allowed the students to "pass" (MG, Interview 1, p. 9). "I guess I am not a big stickler for putting people on the hook. I'll put them on the hook when it's time for me to have time with them, but not in front of other people" (MG, Interview 1, p. 9). Marie said she is particularly good for teaching the low level students because she is also a slow processor and can relate to students who struggle and she has the patience to "tell you the same thing in 25 different ways, if I can think of 25 different ways, and I won't act annoyed by it at all, well I'm *not* annoyed by it at all" (MG, Interview 1, p. 13).

Ultimately, Marie's

> real goal is for you to learn how to learn. … I want them to learn how to be successful in math, but more than that. We live in the 21st century, and this idea of memorizing it, I mean, I just want you to have a, not a procedure but, some confidence in knowing that I can learn things, you know, and that when I learn them, boy, I feel instantly different because I feel better as a person; I feel better about myself. (MG, Interview 1, p. 13)

The case of Marie is especially interesting because Marie used terms and ideas that are consistent with constructivist beliefs and yet was herself very traditional in her approach to mathematics, as were her practices in the classroom. Marie described in detail her own process of change throughout her during- and post-training data. In her second journal, Marie wrote:

> I believe that I am gaining mathematical insight. Most (probably all) of the problems are easy for me to answer, meaning give the proper solution. However, in most every activity I have gained a new and better perspective to help me answer the questions "why" or "how does this relate?" I do not yet feel confident with my new perspective as I am a learner who requires much practice before I have satisfied my questions and concerns. (MG, Journal 2, p. 3)

Marie described this shift in more detail during her second interview:

> I knew everything we did. I had a method and I knew it. I had a method for every single thing we did. So the math did not appear to be challenging to me, unless I took on the task of doing it from a new perspective, which was, you know—and this happened to me and this was the part that was so exciting to me. I did the math the traditional way in the beginning. I'd say almost to three quarters of the way through the course, as a means of knowing what my answer should be when I was struggling with the fraction strips or with the number line modeling or with the grid modeling or—so I did it so that I knew when I started to color in the wrong thing, I knew it couldn't be right 'cause I knew what the real answer was. And maybe you remember that all of a sudden I did some problem and I didn't do that until after and I was like, "I totally get this now. I can do it like this." So it was almost a development. I can't call it number sense because I have a sense of what I'm adding together and multiplying together

> when I do it, in a sense, but it was a deepening awareness of it I think, of what the operation itself meant. Well, remember subtraction? I mean, I could not get to that difference thing and now I love it. But it was just going to be take-away for me. You know? So differing ways of looking at things, and I think it deepened my understanding. I could do it all over again and feel even better about it though, because I don't feel confident in all those new things that I tried. (MG, Interview 2, p. 19-20)

For Marie, learning to understand mathematics in this new way brought a richness to Marie's mathematical understanding.

Throughout the professional development, Marie began to determine that her students needed to have the same sort of experiences with mathematics that she was having. Marie talked, in her second interview, about how the discussions in the professional development about the mathematical practices called her attention to this hole in her teaching:

> My idea of how I was going to make sure these guys understood math, was not really in alignment with those [mathematical] practices. I wasn't allowing them enough… time, room, you know, because I thought, they're going to learn the same way I learned. I'm going to put it all on the board, they're going to write it down, and then I'm going to do 10 problems and I'm going to go, "Step one, write the variables. Step 2, nah nah nah. How do we undo?" And then that's going to make them—not underst—that's going to make them do the math the right way. But I wasn't accounting for understanding or the need, or the enjoyment of perseverance or figuring anything out for themselves, because I was just going to be that person for them. So, those practices and [the instructor] pointing out that—even though it seemed sort of ridiculous or rote by the time—it really was like, "Oh, we did that [mathematical practice], oh, we did that [mathematical practice], oh, we did that, oh, we did that, oh, we did that." And then I thought about my own lessons and said, "Hmm, we don't do that." We need to do a little bit of that, you know? So, I would say that was definitely a big something for me. Very hard to change. (MG, Interview 2, p. 2-3)

Marie began to be purposeful about having students discover mathematical relationships. Marie's second lesson plan included the use of pan balances[42] for solving equations. Marie used pan balance worksheets from the Everyday Math program, but built a discovery activity for students to do prior to using these worksheets. Marie explained why she chose this lesson to submit:

> I selected this lesson for two reasons. First, because I am jumping into pre-algebra for the first time in the sixth grade, courtesy of the new CCSS, and I want the students to have some familiarity with the concept of equality. Secondly, I chose to write this lesson to incorporate some new ideas from UEM; not because I think I should (for Ann's research) but because I feel it is the right thing to do (based on my experience in this class). I want to share some excitement of discovery with my students. I want them to become more literate when describing mathematical concepts and why they work. I want them to be more invested in the process than just a visit with a worksheet and some class notes. I'm hoping for a little more than memorization of the rules! (MG, Lesson Plan 2, p. 11)

In the second interview, Marie talked about the way the lesson helped her students discover the meaning of the concept of equality.

> It [the activity] was excellent for teaching equality. Because an equal sign to them, it's just somethin'. It doesn't really mean—like they have no concept of what that means. … It just becomes language. Everybody gets three plus five is eight, but the idea that well, okay, so if then we took this side and we multiplied by two. Could we double? Could I? (MG, Interview 2, p. 29)

The activity, Marie said, helped students see that equality really meant that their pans were balanced.

Marie had successfully tested her new hypothesis that students can discover mathematical relationships through the use of the standards for mathematical practice and that this discovery process can lead to understanding. Marie experimented with this

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[42] Pan balances are balances with two pans, one on each side of the fulcrum. These can be used in mathematics lessons to illustrate the concept of equality.

idea again in her third submitted lesson plan. This time, rather than creating the lesson from scratch, as she had with her second lesson plan, Marie chose a lesson that she found online. In the reflection on her third lesson plan, Marie described why she chose this particular lesson to submit:

> While investigating our call to mathematical practice standards, it has become clear to me that students should be afforded the opportunity to work with mathematical situations that will allow for true understanding. "My" explanations will likely satisfy many students, but may not inspire them to discover relationships and patterns on their own. The ability to discuss and understand math from differing points of view, I believe is "eye-opening" and beneficial for deeper understanding. It also builds confidence and hopefully enjoyment in the area of mathematics. (MG, Lesson Plan 3 Reflection, p. 1)

Marie described, in her second interview, what it was like to allow her students to "discover/uncover mathematical relationships before I write the procedure on the board" (MG, Lesson Plan 3 Reflection, p. 1). She described how, in her implementation of lesson plan three,

> they did perform their [jumping jacks] and time themselves and that kind of thing and then see, getting them to a unit rate as a result of that, or an expanded rate as a result of that, without giving them any hints that we were going to make them into fractions and make equivalencies or anything, then listening to how they got there. (MG, Interview 2, p. 12)

Marie says she has changed the way she asks students to learn and interact with mathematics and shared an example that was not as elaborate as the changes exhibited in her submitted lesson plans.

> I tell you, I have had some kids that—even as simple as I had them correct a [test-preparation] packet together, but I made the groups intentionally. And if they didn't have the same answer, they had to share how they got it, and they had to convince each other before they could invite me. I have just had some kids that are now teaching. I call them like, 'Professor Cobb' and 'Dr. N.H.' and I mean, I have kids who are just rising to the top with this idea that they

> have really good ideas in their head. I feel awful to say that. Seven years later, Mrs. Gilmore allows children to express themselves. (MG, Interview 2, p. 6-7)

This last sentence sums up the changes in Marie's beliefs: "Mrs. Gilmore allows students to express themselves" (MG, Interview 2, p. 7). To Marie, this means that children need to share in the excitement of discovery and participate in the learning of mathematics.

Leona. Leona Sawyer taught Title I[43] Mathematics to third, fourth, and fifth grade students, which means that she provided small group instruction to students who had not been successful learning purely in the general education setting. Each student worked with Leona two or three times per week for 25 minutes. Some students also worked with Leona for additional intervention through Response to Intervention (RTI).[44] Leona's students received instruction from her in addition to their regular classroom instruction. At the time of the study Leona was in her fourth year of teaching and had been providing mathematics Title I support for each of those four years. Leona majored in human development, elementary education, and special education in college where she participated in a teacher preparation program and earned elementary generalist and general special education teaching credentials. Leona did not have any graduate degrees and teaching was her first career.

Leona hated math as a child, but her feelings towards mathematics began to turn around with her intuitive geometry class in college because she was forced to think about the math instead of just memorizing the steps. In her first interview Leona

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[43] Title I refers to Title I of the United States' Elementary and Secondary Education Act of 1965, most recently revised as part of the No Child Left Behind Act of 2001. Title I provides, among other things, a mechanism for funding of programs aimed at closing the achievement gap between high- and low-performing students.

[44] Response to Intervention is a methodology determining how support is provided for struggling students.

said that she really enjoys teaching math because she feels that "from just teaching it myself I kind of re-taught myself and had a better understanding" (LS, Interview 1, p. 4). She said that by working through textbooks she was using for teaching, she was able to go over the mathematics at her own pace and was able to "get a lot more from it…. Now I absolutely love math" (LS, Interview 1, p. 4), she said.

Despite this love of math, Leona did not have strong confidence in her abilities in mathematics. When asked, in her first interview, what goal she would choose for herself to make her instruction better, Leona said she would like to improve her own confidence in mathematics.

> I've always had that feeling of dread and I'm still getting over that myself. When I'm teaching it, it's no problem; but when you say we are going to do a pre-assessment and a post-assessment, my heart kind of dropped.… I am afraid that I'm going to get it wrong and they're going to call me on it and, I'm going to be so embarrassed, especially now that I am teaching math. So just more confidence with myself in my math abilities and just the knowledge of the math itself because I feel like if I can understand it better than I can figure out how to give it to my students in a way that they need, but if I don't have a good understanding myself then I'm not giving them as much as I could be giving them. (LS, Interview 1, p. 13)

Leona's MKT scores were just above average for an elementary teacher. She scored 0.33 standard deviations above the norm on her pre-training MKT assessment and 1.18 standard deviations above the norm on her post-training assessment. Leona's IRT scores were higher on her post-training assessment than her pre-training assessment. When considered using the standard error of measurement (SEM) of the test, the error bars just barely overlap (see Figure 12). This indicates that there is just under a 68% chance that Leona's true scores are different from one another, a probability that is not typically considered large enough for the difference in scores to be meaningful.

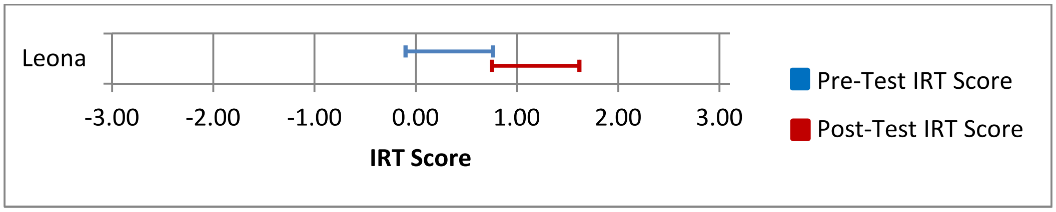

Figure 12. Leona's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

In her teaching, Leona said in her first interview, she wants students to feel as though they can relax and come "out of their protective shell" (LS, Interview 1, p. 7) which helps "build their confidence" (LS, Interview 1, p. 7). She said that it is important to have clear expectations but not "take it too seriously" (LS, Interview 1, p. 7) so that students are not embarrassed to take a chance. Students should be able to experiment with problems and feel like they can get it wrong and make mistakes.

> I just try to make sure that when I am teaching it that the students are not embarrassed to take a chance, that it's a very safe and free environment in a way to where they can experiment with problems and not feel like "Oh, darn, I got this wrong" and "I'm not going to try it again" or "I'm not going to volunteer again." (LS, Interview 1, p. 7)

She wants her students to become more confident learners and "feel more free to have fun and experiment with learning" (LS, Interview 1, p. 7).

Some strategies and techniques work better with some students than others, Leona said. When planning lessons Leona said she tries to do things differently from the way the material is being presented in students' other math class in order to give her students another way to learn the material where they may be more successful.

Leona said she often gave her students something to work towards: points, a game, or individual goals. She believed goals help students become more self-motivated.

> They'll be more excited because they'll say "Oh, I've met four goals so far." I think it boosts their self-esteem and lets them know they can achieve things in learning that they want to achieve. Which for a lot of them, who especially struggle in math, they don't think they ever do accomplish much in math, and that they're more that they are drowning in it. … [When they meet their goals,] they've succeeded instead of failed. (LS, Interview 1, p. 11)

To ensure that students are learning, Leona said lessons should include student independent work. "If they can't do it independently then they probably don't have a really good grasp of it. … I think they need at least an exit slip" (LS, Interview 1, p. 3) to ensure that each student is learning the material and not "rely[ing] on other people" (LS, Interview 1, p. 3) to give the answer and then "regurgitate it back out" (LS, Interview 1, p. 3).

In her first lesson plan and her first interview, Leona talked about the use of discussion with her students. She said she believes that math lessons should involve students discussing the mathematics and explaining what they know. When students' answers disagree with each other, Leona likes to have students participate in what she called "math debates" (LS, Interview 1, p. 2) where each child goes up to the whiteboard and defends his or her solution. Leona believed students should be asked to explain why a strategy did or didn't work. She felt that discussion was often a better use of class time than worksheets. Worksheets, she said, have their place but do not allow for the teacher to know how the student is doing the problem the way discussion does. Leona did feel that independent work was important, but she said that it needs to be balanced with group discussion.

This idea of discussion permeated Leona's data, both pre- and post-training. It is also an area where Leona saw growth throughout the professional development. In her second interview Leona said her beliefs about high quality lessons have not changed, per se, but have been "fine-tuned" (LS, Journal 2, p. 1) She described in detail:

> The biggest thing that I've always liked about or thought about high quality lessons was the discussion piece and I thought as long as they're talking about it [the mathematics] that's good. But going through the class and just seeing the difference between our discussion that we were having about why things were happening and more understanding the whys behind all the rules that we had been originally taught way back when. I realize that discussions that I was having with my groups [of students] wasn't the same and they weren't teaching as much as I'd like the kids to be learning. And so I really started paying more attention to the types of questions that I was asking them to help them kind of figure out the patterns on their own and I just… it was a little difficult at first, but just kind of step back even more because we're used to my particular group of just like, 'Okay, here's how you do it. Now let's talk about it.' And instead I was like, 'Well, here's a problem. Go ahead.' And sometimes we'd have to stop and I would have to intervene and talk about certain things first, but I was very excited to see they were actually able to get started in on their own and they'd start talking about it. It might take a little bit longer for us to get to it because they have to explore it, but they're making the connections and I was like 'This is so exciting.' So just things like that, that I thought I was doing, but then I realized I really wasn't up to par with the types of discussions that they [the students] could be having. (LS, Interview 2, p. 1-2)

Leona said that after the professional development training she had particular goals for her discussions with students above and beyond just having a discussion.

> High quality lessons still need tools, need to be engaging. The kids need to be doing a lot of the talking, but now I'm kind of like it's a [tuning] fork and fine tuned and I have more goals of where I want to bring this or have the students get to. Before I was very happy they were talking about the idea, and then I'm like, "Well, let's talk about the whys more and how did we get here? Can you show me? Where's your evidence? You know, convince me." So I think a little deeper now than before. (LS, Interview 2, p. 8)

In her second journal, Leona provides an example of the way her discussions with students have changed:

> In my lessons, I am trying to get my students to see those connections by asking deeper questions. The other day, I was working on solving word problems. I had planned on asking them whether or not they would have to use multiplication or division to solve, but then I found myself thinking that you could use either because both operations deal with breaking a total amount into groups. So I put my students into two groups, one was to solve the problem with multiplication and the other group was to solve that same problem with division and then share how they used the operation to solve the problem and if which operation they thought would be best to use. At the end I asked them if they could go back in time and change the history of math, would they have split multiplication and division into different operations or would they have combined them into one idea. Their conversation was great. I had some students say that they would combine them because you use the same facts but that they are just switched while other students said that they should be separated because even though they were made up of the same facts, one ended with a smaller number (division) and the other (multiplication) gave you a larger number. They were really thinking about it though and were really getting into it. I just thought that was great. (LS, Journal 2, p. 2)

This heightened focus on the discussion of mathematical ideas in ways that involve students wrestling with the mathematics demonstrates how Leona was able to take her primarily constructivist beliefs and gain skills that allowed her to better implement those beliefs in her classroom.

Leona's reflection on her third submitted lesson plan illustrates how Leona's experience learning mathematics in the professional development impacted her beliefs about quality lessons and her creation of her lesson on addition of fractions. Her lesson involved using pizza to have students discover why $\frac{1}{4}+\frac{1}{4}$ equals $\frac{2}{4}$ not $\frac{2}{8}$.

> I chose this lesson because it took a skill which many of my third graders have such a difficult time really understanding at a deeper level and not just because those are the rules that we are supposed to follow when adding fractions.

> Because they have just been following the rule, they easily forget that rule. This lesson is different, though, because the students are doing most of the discovering and coming up with the "rule" with each other using their pizza slices and their discussions. The students are having fun, which helps them to persevere through the problem solving, too. I also thought about the work that we did on the last class when we were dividing fractions, and realized how abstract that skill still was to me as I struggled to divide some of those fractions. I thought that if I am having trouble with my knowledge of what fractions are and dividing and am still having trouble; I could only imagine what my young third graders are thinking with adding these strange looking number representations. I liked how we had shared different ways to represent the problems, as I found many of them very insightful, but know that left to my own devices, I would have never come up with them. I think the discussions and the shares that we had during class was one of my favorite parts because I was able to look at different concepts in so many different and interesting ways. When creating this lesson, I wanted to give my students that same opportunity, and while I may have to guide more at some points with my particular group of students than [the instructor] did for us, I still think that having the students put their discoveries into their own words to share would be excellent for the others students to share and hear. And just as the manipulatives helped me a lot in the class, I wanted to give the students some tools to use to demonstrate their thinking, and what better tool for a kid than pizza. (LS, Lesson Plan 3, p. 2-3)

Leona began the study as a teacher whose focus was discussion of mathematics topics. She ended the study as a teacher who wanted her students to discover, discuss, and connect mathematical topics. Leona may have eventually arrived at these beliefs without the professional development experience because she already had many constructivist beliefs. When asked about this in her second interview Leona said that the professional development "speeded things along" (LS, Interview 2, p. 5) in terms of her mathematics learning. "Especially in terms of positive, negative integers, fractions." she said, "certain pieces that I don't always get to with my students so I never get the opportunity to look back and re-teach it and relearn it myself" (LS, Interview 2, p. 5). In addition, Leona's descriptions of how her experience as a student in the professional development impacted her planning of her third submitted lesson show that she used the experience of learning to help her decide how to teach.

In her second interview, Leona discussed the changes in her lessons as a result of her experience learning mathematics and her improved confidence in her understanding of the mathematics. The professional development

> made me think about math differently because now all the things I'm bringing up to my kids, especially my 5th graders, we're doing fractions now, and all the things we did with fractions [in the professional development] I thought was really neat because now I understood why you do this with fractions. So I feel like I'm able to talk about it more in depth and have my kids talk about it more in depth because we're doing a lot of the same activities. I feel like I'm more comfortable talking about it because I know the reasons why we're doing it and know the background, if you will, of the concepts and skills a little more. So I do feel more confident and just knowing that I was able to go in and have good conversations with people and bring different ideas to the table, it was very nice to be able to do that. (LS, Interview 2, p. 4)

Leona described building confidence in her own knowledge of the content she teaches to her students and using activities from the professional development in her classroom. When asked, Leona said that she thought her new skills would transfer to content not covered in the professional development. She said her increase in confidence and her new willingness to admit when she doesn't know something helps her learn new ways of approaching content.

Leona used this new knowledge and confidence to change not only her discussions, but her use of instructional time as well. She changed the way she allocated her time both within one class period and across the curriculum with the number of days spent per topic. In her second interview, Leona discussed her allocation of time throughout the school year:

> I think taking the time, I think is something that I've really come to learn. Even though it may take us a whole week to solve a couple problems, it's worth it if they're coming up with a deeper understanding because when they have that deeper understanding, we're going to save time by the end of the year because these things are going to click faster. They're going to have more of an idea of, "Oh yeah. I remember doing this and this is why it happened. So I can try it this way." So just taking that extra time and not rushing the kids through to get where you want them to be, but helping them just kind of get there as they get there. (LS, Interview 2, p. 16-17)

In her second journal Leona provided a specific example of this allocation of time:

> In my class, I have started to spend a little more time with values. For warm-up activities, I have been doing games and questions that ask students to link different (or so we thought ☺) math concepts. For example, my third graders were asked to show me 15 in as many different ways as they could, which led to a great discussion of how addition and multiplication are related with groups and area models, as well as the commutative property, tallies. They were so excited to see how all these ideas meshed together to make a given value. I thought this was a great start for them and hopefully it will be helpful. I know that I only started to make those kinds of connections like we have been discussing in here in college and when I first began to teach and was relearning the information on my own. (LS, Journal 2 Addendum, p. 1-2)

Leona described not only her focus on using more class time to explore values, but also her focus on connecting different representations of values. Although she began to make these connections herself in college, the professional development experience taught Leona to help students make those connections. This idea of connections is a theme in Leona's post-training data.

Leona's pre-training data had grand theme: discussion. Leona's post-training data enlarged that theme to encompass three aspects of lesson quality: discovery, discussion, and connections. All three of these aspects require that the teacher allow the students to participate in the flow of the lesson. Leona discussed this when describing the structure of a high quality lesson in her second interview:

> Of course you can start with "Okay, what is the intention of a lesson? What do we want the kids to get to?" And have your end goal in mind, the end learning piece, but providing the students an activity or problem and then let them take it and go along with it and see where they're going instead of saying, "Okay, well let's do this thing. Okay, now we're going to stop and do this." But letting them just… "Well, okay what do you think the next step would be" and just letting them lead it if they need more time. "Okay, well, we're not going to stop, let's just keep going" or if they get it quickly, then "Let's look at how we use it to solve this problem" or extend it differently. But I think not so much on the fly, you're constantly remembering what your main goal is, but it's making sure that they're using what they need and figuring out what they need to do to bring themselves there. (LS, Interview 2, p. 17)

This new lesson structure, according to Leona, requires the teacher to follow the students' path to get students to their final learning destination, rather than mapping out the whole path for students. This path allows and facilitates students discovering mathematics, discussing mathematics, and making connections between mathematical topics, all ideas that Leona felt, post-training, were critical to quality lessons.

Alex. Alex MacMillan is a certified math teacher for the middle grades (grades five through nine). During the study period, Alex was in her third full year of teaching and taught mathematics to seventh and eighth grade students in a general education setting. Alex also taught science to the same group of students, at times collaborating with another teacher and at times as the primary science teacher. In prior years of teaching Alex had taught math, science, and health to sixth, seventh, and eighth graders. While in college, Alex majored in psychology and minored in statistics. She then earned her Master's degree in Teaching for Social Justice while completing a teacher preparation program. Teaching is her first career.

Alex's pre-training data emphasized her focus on classroom climate and student experience. She described wanting students' education be meaningful to them rather than having “teachers talk at you, the education comes at you” (AM, Interview 1, p. 8). She believed that all lessons, math or otherwise, should help the student develop as a person, not just to learn mathematics. She described the mathematics content as “the parsley” (AM, Interview 1, p. 11) on the math lesson dinner plate. Alex identified students as the most important aspect of a lesson and the “steak” of the math lesson as “teaching them how to learn, rather than just teaching them this stuff” (AM, Interview 1, p. 11).

Part of teaching students to learn, according to Alex, meant teaching students to be members of an academic community “where discussion is okay, condescension is not, rudeness is not; where you can always ask somebody to clarify more” (AM, Interview 1, p. 8). She expressed holding herself to these same standards. Alex developed verbal cues for students to use to let her know when they were confused or when expectations were unclear. She discussed wanting students to have some measure of control over their own learning and to be able to get help when they were feeling lost. “This is a community,” she said in her first interview, “and I am their teacher, but I also really care about them, and who they are as people” (AM, Interview 1, p. 8).

Alex’s grading practices reflect her beliefs about how students learn and their role in their own learning. Alex said she grades homework based on the effort shown to complete the assignment, not based on whether or not it is correct. In her first interview Alex said she would never give a student a “bad grade for going home, working hard on your homework, and doing it wrong” (AM, Interview 1, p. 5). Alex shared assigning grades of A, B, or “not yet” (AM, Interview 1, p. 9) on assignments. If the student had not earned an A or B on the material, Alex said she would ask the student to redo or revise the work until the work reached a satisfactory level.

Alex expressed wanting to develop “persistent problem solvers” (AM, Interview 1, p. 8). She shared that she has told her students that “smart isn’t what you are, it’s what you do” (AM, Interview 1, p. 10). She expressed wanting to build students’ confidence along with their skills and provide avenues in the classroom to meet students where they are and build their ability to reason and discuss and problem solve. The true mark of a quality lesson, according to Alex, is in the answer to the question: “Does your lesson accomplish these objectives in a way that is thinking about the kids and what thinking about what those kids need?” (AM, Interview 1, p. 17). This question highlights Alex’s focus on students as people and as learners of mathematics.

Alex discussed the structure of her daily lessons in her pre-training interview. Her typical lesson, she said, begins with a warm-up that introduces students to the topic, followed by a homework check and housekeeping, the main lesson that is related to the warm-up, the assignment of homework, and an exit ticket. The exit ticket is a short assignment that students complete before they leave the room. Typically the exit ticket is used to verify that students have learned a particular aspect of the lesson.

In her first interview, Alex shared that she would soon begin teaching an enrichment session with some of her students in which she intended to have students “do an investigation and either generate data or look at patterns and figure what we can see here, and summarize and refine and state as simply as possible, and then have kids write about it and talk about it” (AM, Interview 1, p. 15). This focus on problem solving and reasoning is clear across Alex’s pre-training data. In her interview, she expressed wanting to find ways to include more practice into her lessons in a meaningful way.

> I have a tendency to try and go for higher-level in-depth thinking problems. … I want to get better at finding meaningful ways to bring that stuff [practice] in and not have it necessarily be a big discussion, but have it be some meaningful practice. (AM, Interview 1, p. 13)

Alex thought of this focus on practice as a characteristic of many mathematics classrooms, but believed that the problem solving and reasoning was a more important aspect of mathematics learning. "I want to prepare them for the world that they live in" she said when discussing computational practice, "and also help them to be part of the change to the new world that I want to be in" (AM, Interview 1, p. 14), referring to the amount of problem solving and discussion she used in her classroom. "I like it when they can discover rules and figure out the structure on their own and I can say, "Yeah, and then this that you've identified is called this and this is like this," but I really love it when they can produce and I can help them label" (AM, Interview 1, p. 4), Alex said. She went on to say that "sometimes you do need to teach them something about the structure and the format" (AM, Interview 1, p. 4), but that these classes are "rather dull and boring" (AM, Interview 1, p. 4).

One theme for Alex that began in her pre-training data and grew throughout the data collection is use of simple problem contexts that allow students to explore deep mathematics. The simplicity of context, Alex suggested in her first interview, allows the students to focus on the mathematics in the problem. Alex's first lesson plan was chosen because of this feature. In the interview, Alex described why she thought this was important:

> I think that one of the things that I have seen happen is there will be either problems that are too complicated, or too convoluted … [where] you can get too much into the details, and you've got all these trees, and you cannot see what in this forest, right? Or there are all of these problems, and you see the format, and you're like, "Okay, that is the slope, and that is the constant," and you have no real concept of what it means. (AM, Interview 1, p. 2)

Quality lessons, Alex wrote in her second journal entry, "should be relatively simple in overall structure (of the *lesson*) but that the tasks should be worthwhile and require students to do the heavy lifting when it comes to making sense of the problems and their solutions" (AM, Journal 2, p. 2). Alex stated that she struggled to develop a repertoire of problems that included both a simple context and deep mathematical meaning, but expressed actively attempting to do so.

This belief was still evident at the end of the study when Alex expressed liking

> those self-reinforcing problems that highlight the mathematics so that you don't have to know tricks; you don't have to have caught that one special word in the beginning…. you can figure it out if you stick with it, if you work through it; and that there are mathematically important things along the way. (AM, Interview 2, p. 28-29)

In these problems, she said, there is a "series of discoveries [and] you have a small reward at the end of each one. You're like, 'Oh, okay, so now I get this. Now what's next?' … It gives you opportunities. It makes you want to keep going" (AM, Interview 2, p. 28). Alex noted that she saw the same value in the course of the professional development when teachers were asked to work on "a series of problems, like the integers, adding a series of a group [of] six or seven problems that were meant to highlight some mathematical realization, some mathematically important thing about them. That was really key" (AM, Interview 2, p. 29).

Alex's beliefs were consistent with the constructivist end of the spectrum. Nowhere is this more apparent than in her description, from her final interview, of why it is important for students to work through problems that are simple in structure but encompass deep mathematics:

> Those things that we create, and that we bring meaning to ourselves, stay with us. And that those things that we're led to understand or that are too constructed for us, we don't really hold onto because we don't make our own meaning out of it. (AM, Interview 2, p. 26)

Alex's beliefs about the nature of quality mathematics lessons did not change through her participation in the professional development. Rather, she felt that she experienced her beliefs about lesson quality in action through the professional development. In her final journal, Alex wrote:

> My understanding of what a high quality math lesson is has not really changed, but I think that my ability to design lessons that are high quality has greatly improved. I have seen my beliefs confirmed and reinforced by the practices and structure of this class—especially the modeling and discussion. (AM, Journal 3, p. 2)

Alex's own constructivist beliefs were reinforced through her participation in a professional development training that employed constructivist methods. This allowed her to see constructivist teaching in action and learn not just mathematics, but how to teach mathematics.

Alex scored 2.68 standard deviations above the norm on her pre-training MKT assessment and 2.05 standard deviations above the norm on her post-training assessment. Although Alex's IRT scores were lower on her post-training assessment than her pre-training assessment, when considered using the standard error of measurement (SEM) of the test, the error bars overlap (see Figure 13). This indicates, especially in conjunction with the known ceiling effect of the assessment, that Alex's true scores on the test are within the margin of error of the test and the difference in scores is not meaningful.

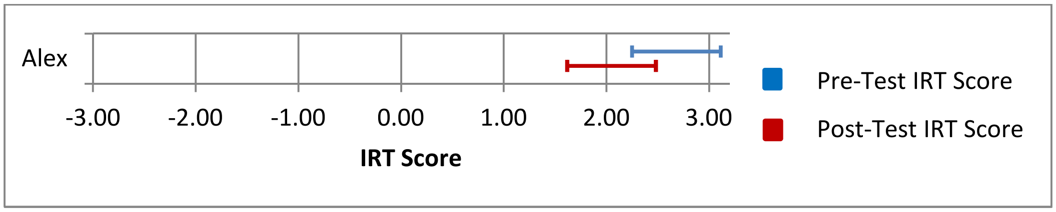

Figure 13. Alex's pre- and post-training scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175.

Despite this lack of growth in knowledge as measured on the MKT measures, Alex is clear about her growth in content knowledge as a result of her participation in the professional development. In journal two, Alex gave a specific example of a mathematical idea that was new to her:

> I've been thinking a lot about the progression from place value in base-ten, to place value in other bases, to the meaning of the value of various fractions. I really never learned (or never really understood) the connections, and I think that this connection is still missed by many (most?) teachers and students. It seems so obvious now, and it makes reasoning about computations both more meaningful *and* much easier. Crazy to think I have never had any instruction in different bases and had never really thought about the connection between place value and the number of pieces that it takes to "carry over" to the next place value, and/or to make up one "whole"—it seems impossible to really get the idea of value/place value without it! (AM, Journal 2, p. 3)

In other topics, Alex felt that she gained depth of understanding. "I learned a lot more" (AM, Interview 2, p. 9), she said in her final interview.

> Whereas [before the professional development] I thought: "Yeah, I know dividing fractions. I know how to do it." And if I was going to rate myself on a scale of one to five how well do you know this topic? I would have said five.

> Now I would say like three point five, because I know it really well, but I actually can't explain it that well. And I realized how much more practice I need at explaining. (AM, Interview 2, p. 9-10)

Later in the interview, Alex discussed her gain in knowledge again:

> What I've found in the class [UEM] was that all of the different ways to understand the problems we were doing really lead me to a much richer understanding of it. … I guess if you were to visualize my understanding of division it went from being this very Charlie Brown Christmas tree looking thing, into a much bigger, more filled out, more elaborately decorated Nutcracker type of Christmas tree. (AM, Interview 2, p. 14)

In this last statement, Alex is attributing some of her gain in content knowledge to the experience of being in the professional development and hearing the different ways her peers solved problems.

That experience of learning mathematics influenced Alex's beliefs about pedagogy as well as content. Alex wrote about this in the reflection submitted with her third lesson plan:

> The lessons I have chosen to submit have not changed much over time, but the way I implement them has. The things I have learned from our professional development about the structure of a class that focuses on student engagement and discovery--the level of difficulty, the number of problems, and the amount of student interaction--has improved my thoughtfulness and patience for these elements in my own classroom. It is really true that when students can generate the knowledge, they have more ownership of it. Reminding myself to pare down lessons to the essentials is the most difficult part, as is facilitating classroom discussions for a diverse population. Breaking the larger class down into small groups has really helped this. I think that the greatest lesson I have taken away from my experiences in the Elementary Mathematics course is that the more students are allowed to engage with the material and make sense of it themselves, the better and more flexible their understanding of that material. (AM, Lesson Plan 3 Reflection, p. 2-3)

This reflection shows not only Alex’s change in beliefs about pedagogical strategies appropriate for teaching mathematics, but also that Alex has tried to implement these new beliefs in her classroom. This was also evident in Alex’s third journal:

> I have made a lot of changes to my instruction since the beginning of this course. I allow a lot more time for students to discuss problems, and have begun providing challenge problems so that I can allow discussions to go on for long enough for everyone to say what she or he needs to, and also provide a mathematically enriching option for those who feel that they have finished their discussion or exploration of a problem before others. Second, I have moved to designing my own lessons (sometimes I still use problems from our text for practice once we’ve explored a concept or for exploration). My experience in this class has shown me that learning can be more meaningful when we engage with and show or prove the why’s of mathematics (not just they how’s). Much like this class [UEM], I now use fewer problems, provide time and activities to explore and represent the problems in different ways, and make sure that we discuss our thinking afterward. In this class I have found that by listening and working to understand the thinking of others, I gain a better understanding of my own thinking. I have also found that it really helps to compare and contrast our ideas. When I use this method in my own classes, I find I am often surprised by the eagerness of students to share their answers and ideas, and their ability and willingness to critique the reasoning of others. I am still new at this kind of teaching and I know it will take a lot of refinement to make it safe and productive for all students, but from my experience in this class and the changes I have already seen in my students, I know that the more we practice the more deeply we will learn. (AM, Journal 3, p. 1-2)

Alex attributed these changes in pedagogical beliefs and practices directly to her experience in the professional development. The experience was meaningful to her both in her role as student and her role as teacher. In the second interview Alex said,

> The class gave me so many things to think about, really lasting memories too; really definitive memories from the classes about what we were doing and the way that felt to be doing those things, solving the problems. In some ways I wish had more of an experience of what [the instructor’s] experience was. There was a point in the class when I started to try and watch her; what is she doing while we’re doing this? That’s going to be my role, so I was watching her. (AM, Interview 2, p. 11-12)

Alex began the study with constructivist beliefs about the ways in which mathematics lessons should be taught. Her experience (a) learning mathematics with more depth, (b) seeing the various ways teachers solved problems, and (c) observing the ways in which the instructor structured and taught the professional development reinforced these constructivist beliefs and helped Alex see ways that she could implement these beliefs more effectively in her classroom.

Eight case studies, many stories. These eight white women entered teaching at various times in their lives and followed different pathways to get to the classroom. All did complete a teacher preparation program and taught in the elementary grades. Some worked exclusively with children identified as having special needs, others worked with the general population. Most taught in a brick-and-mortar school building, but one taught in a virtual environment. Some loved math as a child, some hated math. Some were confident in their abilities as math teachers, others sought out training to improve an area of weakness. Their classroom experience varied, but all were experienced teachers.

Each case is, in and of itself, a rich description of one teacher's journey through learning. In examining each teacher's data, I tried to find the essence of what was most important to that individual in both the pre- and post-training phases of the study. These overarching themes are encapsulated in Figure 14. This visual became a quick, easy way to highlight the changes within each individual case and to illuminate shifts in these themes as the study progressed. These eight women experienced learning in a unique way. Each story provides a description of that learning process. Taken together, these cases provide replication and can lend insight into teacher learning as a whole. This process is considered in the next section.

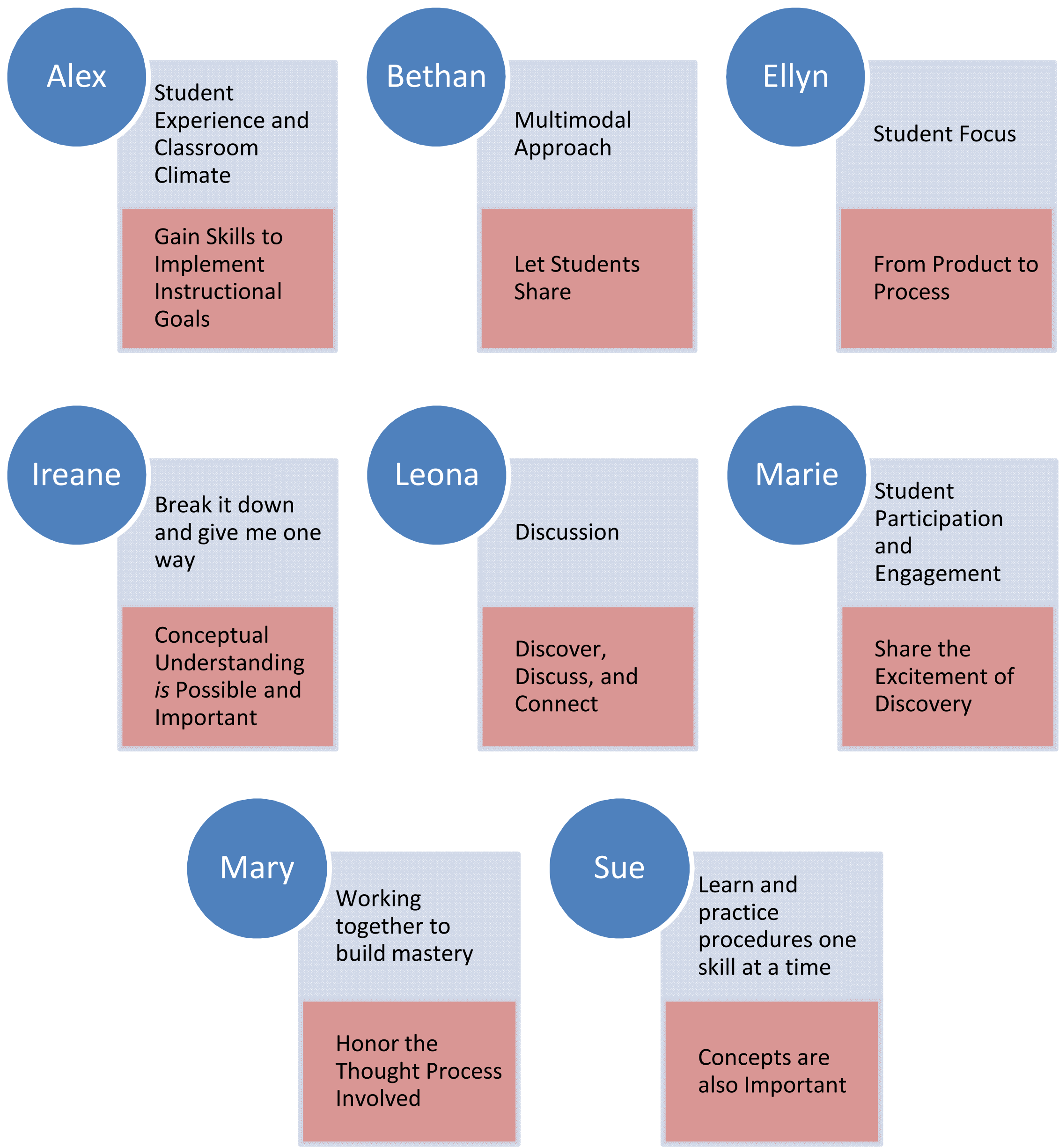

Figure 14. Snapshots of Teachers' Pre- and Post-Training Beliefs. The areas in blue give the overarching themes in each teacher's pre-training phase belief statements, while the areas in red give the new overarching themes in each teacher's post-training belief statements.

Cross-Case Analysis

This multiple-case design allowed me to gain a holistic view of multiple journeys through the process of change. Comparing and contrasting the eight individual cases allows for the examination of trends in teachers' experiences. The replication that multiple cases provide allows for the cross-case analysis to be explanatory, not just descriptive (Yin, 1994). The diversity of the eight cases increases the likelihood that trends from this analysis will be important for other, similar situations (Patton, 2002). The cross-case analysis revealed themes cutting across three broad areas: teacher beliefs about quality mathematics lessons, teacher learning, and teacher beliefs about themselves. Each of these areas is discussed in detail below.

Teacher beliefs about quality mathematics lessons. I set out to study teachers' beliefs about quality mathematics lessons. Although I was able to gather data and examine some of my initial study questions, I was surprised by some of the unexpected places my data led me. First, teacher beliefs were broader than I anticipated. I expected to study two types of teacher beliefs: beliefs about mathematics, and beliefs about pedagogy. Teachers' belief statements required me to broaden my view and also examine teachers' beliefs about students. Second, the definition of quality that I chose from the literature was too focused and not useful for analysis of teachers' belief statements. Third, teachers considered lesson plans to be very different than lessons themselves. These three surprises required me to adapt my research questions and methods of data analysis to ensure that teachers' beliefs were accurately represented. These changes are discussed in more detail below.

This section follows my analysis of teacher beliefs. Because I analyzed data as it was collected, I watched teacher beliefs develop and change. This discovery process was not as linear as it appears. I have tried to make it understandable while showing the back-and-forth nature of my discoveries. I have also tried to balance reporting the differences in the data as well as the similarities. The differences are important because they help us understand the variety of responses among diverse individuals. The similarities are important because they show us what experienced math teachers believe and how those beliefs change.

The introduction of beliefs about students. Data collection for this study began in January of 2013. Sue and Mary were my first two interviewees, both at the end of January. Both teachers discussed students in their interviews, but in ways that related to the teaching or learning of mathematics. Sue discussed how the self-paced nature of the online environment was beneficial to her students because they could review, repeat, or take breaks from their learning and not have to "play catch-up" (SD, Interview 1, p. 6). She talked about the need to personalize lessons with examples and real-world applications that students could relate to. She talked about the importance of laughing with students and making learning fun. Sue's pre-training data was in many ways exactly what I expected to find; it was focused on the mathematics and mathematics lessons.

The main theme of Mary's first interview centered on the teacher and student working together to build mastery of mathematics skills. This team approach, Mary said, was important to building trust with students so that they knew she would not "nail them for not getting something" (MW, Interview 1, p. 10), but would work with them to help them master the material. She discussed individualizing the instruction for each student so that each student worked on the skill he or she needed to master next. Although Mary's beliefs were very student-focused, they were focusing on student learning and mathematics teaching. Again, I could fit these beliefs into my schema of mathematics lessons. Because both Sue's and Mary's ways of discussing students also encompassed mathematics or teaching, I interpreted these beliefs as beliefs about mathematics and beliefs about pedagogy.

It was not until I interviewed Alex beginning of February that I was forced to confront my own ideas of what quality mathematics lessons would entail. The following excerpts from Alex's first interview show not just Alex's focus on students and the classroom experience of students, but also my attempts to lead Alex to discussions that were more focused on the mathematics. Alex was having none of it. Although she did answer my questions, it quickly became clear that she wanted and needed to talk about students as students. To Alex, answering my questions was not possible without this discussion of students. Once I realized this and allowed Alex to express her beliefs about students, I was also able to learn quite a bit about her beliefs about mathematics and pedagogy as well. Emphasis is added to the excerpts below to highlight Alex's focus on students despite my attempts to discuss mathematics lessons.

> Q: So, it sounds to me like you have really a lot of goals for yourself in terms of your math instruction? And you want –
> ALEX: Oh, yeah, like here. [Holds hand above head.]
> Q: Yeah. So, tell me about some of those. Where would you love to be if it were a perfect world, and you were super human?
> [Later….]
> ALEX: So, I don't know if that makes sense, because I have not really outlined like specifically like what are my goals?

Q: No, but that does make sense. And so, a lot of what you've said is really – it sounds trite to say, "classroom climate stuff" –

ALEX: Yeah.

Q: – but it kind of is.

ALEX: No, it is; it totally is.

Q: It's about the learning environment as a whole.

ALEX: Yeah.

Q: So, now I want to take you down the road of more mathematics instruction.

ALEX: Persistent problem solvers; patience, not necessarily patient, but persistent. …

[Later….]

ALEX: So, ideas of ability versus effort, and all that. And so I'm reading a lot about it, and I'm trying to figure out how to help them there, but a lot of them are so smart or, if I don't understand it, it means I'm not smart. If I fail the test it means I'm not smart. They're really ability-oriented, and I want to figure out how to get towards effort-oriented, *and I know we just moved away from talking about math again, but* – [laughter] – it's all about –

Q: It is.

ALEX: – this is how the math; *the math is like the small stuff*.

Q: Yep.

ALEX: It's really –

Q: Yeah.

ALEX: – the like, I don't know, *if you had a steak, and potatoes, and peas, or something, I don't know, little fringes. It's like the parsley.* [Laughter] *The math is like the parsley, and this is the stuff that's really, really important*.

Q: Sure.

ALEX: And we could be talking about anything, and they can handle it.

Q: Yep, yep.

ALEX: And that's the key, so –

Q: So, although you say that, and I get where you're coming from there, because all of this stuff about the people; it's the person learning the mathematics, and so if you know the person –

ALEX: Yeah, yeah, yeah, *and teaching them how to learn, rather than just teaching them this stuff*?

Q: Yes, yes.

ALEX: But –

Q: But from what you're describing about your math classroom, it sounds very different than the typical math classroom, I think, still today?

ALEX: Um hmm. [Indicates, "Yes."]

Q: And you've made purposeful choices to make it that way.

> ALEX: And it's not always that way, like *I have to meet them where they are*. (AM, Interview 1, p. 6-12; emphasis added)

Before interviewing Alex, my attention was focused on my interview questions from the interview protocol (see Appendix G).[45] When teachers wanted to talk about things that I considered slightly "off-topic," I let them talk until they paused and then asked them a question from the interview protocol. During Alex's interview, my realization was limited to the idea that I had to allow Alex to talk about what she needed to say about students in order for her to express her beliefs about quality mathematics lessons. After interviewing Alex, I made sure to ask all of the questions on the protocol, but was less worried about getting to them and thus less concerned with refocusing the teachers on the interview questions. Instead, I provided more wait time and encouraged teachers to keep talking, even if it was about something that I considered off-topic. Only when teachers appeared ready to move on did I ask another question from the protocol. I did not expect to find the off-topic data useful in answering my research questions, but thought that it was more effective at allowing teachers to express their beliefs.

It was only as I was coding Alex's interview that I began to realize that I needed to broaden my view of my study to encompass teachers' beliefs about more than the teaching of mathematics.[46] This realization that I needed to broaden my view of my study is evident in the following excerpt from a memo written as I was wrestling with how to approach the scope of my findings:

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[45] Only Sue and Mary were interviewed before Alex. All other teachers were interviewed after Alex.

[46] I coded Alex's interview after I had completed the first round of interviews with all eight teachers. Therefore, this realization did not change how the first round of interviews was conducted, rather how I thought about the importance of various codes and categories in answering my research questions.

> When coding Alex's first interview, I realized that I have two categories (maybe more) of data about quality lessons: the classroom climate type stuff, and the mathematics lesson stuff. I want to focus on the mathematics lesson stuff, but the classroom climate piece is certainly important, also. Maybe it should be a separate section within the dissertation, more as part of the teacher identification of quality piece rather than a part of quality math lessons. Maybe it shouldn't be in the dissertation at all? I'll have to see what my "story" is for my dissertation and see how it fits in. [Here I was referring to the story the data tells about my research questions, rather than topics unrelated to my research questions.] …
>
> Alex gives me the tie-in: The math is the parsley, the teaching students to learn is the meat and potatoes. To take that analogy further, the meal is incomplete without the parsley. Recognizing how the parsley fits into the meal is important, but this dissertation is primarily about how we work with that parsley. (Gaffney, Climate versus Lessons Memo, p. 1)

I was stuck on studying the small slice of teacher beliefs that had to do with mathematics specifically. Over time, as I worked more and more with my data, I began to realize that I was studying not only teacher beliefs about quality mathematics lessons, but also teacher learning of mathematics. In examining teacher learning in the literature I found models of teaching that included content, pedagogy, and students that characterized what I was seeing in my data. Once I broadened my view of my study I was able to see, through Alex's interview, that my notion of what I was studying was limiting me. In order to be true to teachers' beliefs about mathematics and teaching, I had to consider these beliefs in light of teachers' beliefs about students as well.

Pre-training teacher beliefs. Teachers' initial belief statements, made up of data from their first lesson plan with accompanying reflection and their first interview, shared some common themes and highlighted other differences.

Five teachers (Ellyn, Ireane, Leona, Marie, and Sue) believed it is important to provide step-by-step procedures for students to follow when solving problems. The

Special Education Teachers (Bethan, Ellyn, Ireane, and Mary) talked about individualizing content for students. Some discussed individualizing in terms of what is taught, while others discussed individualizing how content is taught. Three teachers (Mary, Leona, and Ireane) shared that they were not confident in their own mathematics ability and were looking forward to gaining content knowledge.

Issues of classroom climate and the emotional safety of students were prevalent in the data from nearly all participants. These included building students' confidence or comfort level when working with mathematics (Alex, Bethan, Ellyn, Leona, Marie, and Mary), limiting student anxiety (Alex, Ellyn, and Ireane), and building trust with parents (Ellyn) and students (Leona, and Mary). Sue did not bring up issues of emotional safety or climate in her pre-training submissions or interview. This does not mean that these issues are not important to Sue. It may mean simply that my questions did not specifically ask about these issues (see Appendix G) and Sue viewed such answers as beyond the bounds of my study. Sue would have been in good company; I had not yet broadened my view of what I was studying to include areas beyond mathematics.

All teachers discussed wanting to engage students, although the methods by which they suggested engaging students differed. Bethan discussed using a multimodal approach so that students were actively moving their bodies during an activity. Ellyn proposed using fun activities that tie into student interests. Marie and Sue emphasized making connections to the student's world experience, while Ireane said that tying the lessons into money and food make it easier for students to relate to the lessons. Mary said she uses a team approach to engage students in the task of finding what they do not understand and working to remediate those issues until the skills are mastered. Leona and Alex emphasized students engaging in discussion to explain the mathematics while Alex was alone among the teachers in that she discussed students developing and discovering the mathematical ideas contained within tasks.

Issues with MQI coding. When coding teachers' pre-training data, I coded according to emergent themes and also according to the dimensions of mathematical quality of instruction (MQI). During the coding process, I found that I was able to code lesson plans and teachers' reflections on lesson plans according to these dimensions of quality, but that these codes were less suitable to other types of data. Teachers, in their journals and interviews, discussed aspects of quality, not lessons themselves. The MQI dimensions describe features of lessons. Rarely would teachers discuss the mathematical features of lessons, even when discussing lesson quality. When teachers did discuss areas that were addressed by dimensions of MQI, I found that the codes according to teacher statements better captured teacher beliefs than the MQI codes. The MQI codes were too narrowly focused on the mathematics and were not broad enough to encompass many teacher beliefs about pedagogy or students.

In examining the data that was coded according to the dimensions of MQI, I found little of interest, and became concerned that analysis that followed this method would present flawed findings. This does not mean that MQI is flawed, but rather that my data was broader than MQI and so it was a poor match for my data. I was using MQI not as a rubric by which to rate a lesson, but rather as a way to classify characteristics of lessons and teachers beliefs about those lessons. This use was not the intended use. When used according to standard MQI procedures, the rating scale uses interval coding to provide some measure of how strong a lesson is according to each dimension. By attempting to use the MQI as a checklist of sorts, I had removed this measure of strength and turned the rating scale into a checklist. This checklist no longer provided enough differentiation among lessons.

A larger issue with using the MQI rating scale to rate lesson plans was that teachers viewed their lessons as distinct from their lesson plans. In her second interview Ellyn said, “for me there's a difference between what's the lesson plan and then the lesson itself. So, I feel like [the change is] more my lesson, the actual instruction than the lesson plan” (ED, Interview 2, p. 1). Ireane expressed similar sentiments: “I keep thinking of lesson plans and I know you’re not thinking of that but a quality mathematics lesson” (IC, Interview 1, p. 12); as did Bethan: “I do not think the lessons have changed. The lessons are basically the same. Presentation would change more than the lesson itself” (BO, Lesson Plan 3, p. 5). The fact that teachers did not see the presentation of their lessons as part of a lesson plan made it unlikely that an analysis of the lesson plans themselves would accurately represent their goals for classroom instruction. For this reason, teachers’ lesson plan submissions were more useful as a springboard for discussion about what lessons did or should look like rather than as informative data about the instructional goals and strategies they would use in particular lessons. As a result, I stopped thinking of my second research question (When these teachers submit lesson plans for quality mathematics lessons, what instructional goals and strategies do they propose?), as a separate question but rather used lesson plans as a tool by which to gather data about the other research questions.

During- and post-training teacher beliefs. As teachers began the professional development training, teacher beliefs began to change. The speed and extent of these changes varied greatly. This section provides examples of changes in beliefs that illustrate the nature and process of teachers' changing beliefs and culminates with three major shifts in teacher beliefs.

Teachers submitted their first journal entry after the third class of the professional development. In nearly all cases, teachers' beliefs about quality lessons had not changed. Two incidents stand out. Sue wrote about her struggle to move beyond the procedural answer for the problems posed:

> Today's instruction was based on being a student and exploring to find the answers to the mathematical questions posed by the instructor. I found this exceptionally hard because in my math instruction from 1963-78 was based on memorization and rules. My current teaching assignments have given me prior knowledge to the questions asked and it was hard to pretend I didn't know. However working in groups helped me to explore alternative ways and because the others also teach middle school we could talk about how some of our students would respond or nitpick the question. (SD, Journal 1, p. 1-2)

Sue recognized that she explored alternative ways to solve the problem but does not appear to find value in that activity. Contrast this to Ellyn's response:

> We have had 3 classes at this point, and I have already changed how I view a high quality math lesson. I have changed my teaching methods to include student explanation of what they understand about the lesson. I welcome thinking aloud and demonstration of understanding through drawing. For example, one of my students thinks aloud as she is working on math problems. I have felt a bit unsettled that she is talking aloud while working, perhaps distracting the other student in my small math group. She also prefers to draw representations of the math problem (ex. division of fractions), and I have wanted her to focus on the calculation steps. Today, she was taking a test, and for one of the word problems on the test, she did not do any calculations, but verbally reasoned the problem as I listened. She achieved the correct answer by describing how to do it. I gave her full credit even though there was no

> written work beyond the answer. She understood it, and I knew she understood the concept, which was the point of the lesson. I also am more appreciative of her need to represent math problems through drawings, and will encourage her to do so more as that is how she best thinks mathematically.
>
> The class has changed my feelings about a high quality math lesson as being something the teacher provides to the students while they listen, then practice on their own. I am currently thinking of a high quality lesson as allowing the student to engage with a concept or problem in order to discover their own answers. Teachers are there to scaffold that learning process, and guide the students as they are discovering. (ED, Journal 1, p. 2-3)

Ellyn changed her focus from one of procedure to one of conceptual understanding. Ellyn was the first teacher to express this shift, but others expressed similar changes in beliefs as the study continued. Ellyn also discussed the idea of engagement in terms of the student's engagement with the mathematical concept and the process of discovering mathematics with the teacher as a guide in this process. These ideas also permeated the data by the end of the study.

Teachers submitted their second journals and lesson plans after completing approximately half of the professional development. Many teachers discussed gaining content knowledge in that they were beginning to understand why particular procedures worked and connection between and among mathematical concepts. Marie expressed this in her journal:

> I believe that I am gaining mathematical insight. Most (probably all) of the problems are easy for me to answer, meaning give the proper solution. However, in most every activity I have gained a new and better perspective to help me answer the questions "why?" or "how does this relate?" (MG, Journal 2, p. 3)

These new understandings are evident in teachers' statements about how their beliefs about quality lessons have changed. In her second journal Alex wrote:

> I can say with confidence that experiencing this kind of professional development has led me to reflect in more structured and specific ways—not so much *what* I am teaching (my classes are studying topics that are a bit beyond the scope of this professional development), but *how* I teach. (AM, Journal 2, p. 3)

Other teachers, even those teaching content directly related to the content of the professional development, expressed similar sentiments. Leona wrote about refining the types of questions she asked her students in order to get at these connections between mathematical topics. Bethan described seeing many ways to solve a problem and her new willingness to have students discussing their methods and sharing them with each other.

Marie, in almost an echo of Ellyn's earlier statement, described a change in the way she defines the term engagement:

> UEM has enforced my belief that students need to be engaged in a lesson. However, the definition of engagement is emerging a bit differently. Originally, I would consider engagement to be evident if students were participating, active, and productive. I now consider that their participation should reveal engagement in the standards for mathematical practice as set forth in the new Common Core State Standards. I want to see perseverance, abstract reasoning, viable arguments, modeling, precision, structure, patterning…….not because it is required of me, but instead because it shows ownership, deep thought, and life-long learning skills. (MG, Journal 2, p. 1)

Despite professing these new beliefs, Marie was still uncertain as to how this new level of engagement would work in a classroom of students. She wrote:

> I am not yet making changes in my classroom. I feel confused by how to present this information to 6^{th} grade students. I do not yet know if they are capable to explore the concepts (or willing to take risks) that this room full of adults has taken. I feel like my students would be intimidated by learning that requires this much perseverance. (MG, Journal 2, p. 3)

It was not long before Marie was convinced that her students could indeed rise to the challenge. For her second lesson plan submission, Marie selected a lesson that required students to discover the rules of equality for solving single step equations. In the procedure she wrote: “Try to move children into these discoveries without telling them the expected outcomes first” (MG, Lesson Plan 2, p. 1-2). Marie described creating

> this lesson to incorporate some new ideas from UEM; not because I think I should (for Ann’s research) but because I feel it is the right thing to do (based on my experience in this class). I want to share some excitement of discovery with my students. I want them to become more literate when describing mathematical concepts and why they work. I want them to be more invested in the process than just a visit with a worksheet and some class notes. I’m hoping for a little more than memorization of the rules! (MG, Lesson Plan 2, p. 11)

Marie used the lesson and felt that it helped her students understand the concept of equality and the rules for solving equations.

Teachers submitted their third journals and lesson plans at the end of the professional development. For her third submitted lesson plan, Marie submitted a lesson that asked students to do jumping jacks and time themselves. Students then created a rate, such as jumping jacks per minute, and used this rate to find unit rates and solve proportions. On its face, this lesson involved less mathematical involvement than her second lesson plan, but Marie described implementing the lesson in a way that had students describing their thought processes as they solved proportions and explaining their methods to the class. She described the three different methods that her students developed for solving proportions. In the past, she said, she would have taught her students the process for cross-multiplying, they would have practiced it, and they would have moved on. What has changed for Marie is that she “is going to add a piece to the middle now that lets them explain, talk about, note the structures, note the connection[s]" (MG, Interview 2, p. 46) in the mathematical content.

Marie's third lesson plan is one example of how teachers changed their implementation of their lessons. This change in implementation was often absent from the lesson plans. This notable absence is one reason why I discarded my analysis of lesson plans as a separate research question and instead used the lesson plans as a way to gather data about my other research questions.[47]

Marie's process of change was not atypical; it shares many characteristics of change that other teachers also discussed. What is unique about Marie's change process is that every data collection phase showed a distinct phase in Marie's journey. Although other teachers had similar journeys, their path was not always as clear or as dramatic.

In teachers' post-training data—their third journals, third lesson plans, and second interviews—it became apparent which of their beliefs had remained the same and which of their beliefs had changed. Teachers still believed many of the same things they had expressed at the beginning of the study. The teachers still said that the classroom climate was important and that students' individual needs must be met. They still believed in providing time for students to practice procedures and to encourage proper use of vocabulary.

I noticed that in describing their beliefs about quality lessons, the teachers were rarely saying that their beliefs changed, even though I thought they had, rather they

[47] See also the Issues with MQI coding section in this chapter for more discussion on this decision.

used terms like "refined," "deepened," "widened," or "reinforced."[48] This indicates that teachers' beliefs had, like Marie's meaning of engagement, become more specific than they were before. This specificity involved three main shifts. Teachers' beliefs became more focused on mathematical reasoning, more focused on inquiry, and more student-centered. Initially, I only saw these as changes in beliefs. I only later realized that these three shifts directly correspond to NCTM's (2000) three categories of teacher beliefs: beliefs about mathematics, beliefs about pedagogy, and beliefs about students; and also correspond to the three circles of knowledge about subject matter, teaching, and learners in the framework for understanding teaching and learning (Darling-Hammond & Bransford, 2005).

More focused on mathematical reasoning. Instead of wanting students to be busy, active participants, the teachers now wanted their students to engage in meaning-making about mathematical concepts and ideas. Marie's changing definition of engagement discussed above is one example of this shift, but other teachers also experienced shifts in their beliefs toward lessons that are more focused on mathematical reasoning. Alex wrote about this engagement in mathematics as part of the reflection on her third lesson plan:

> I think that the greatest lesson I have taken away from my experiences in the Understanding Elementary Mathematics course is that the more students are allowed to engage with the material and make sense of it themselves, the better and more flexible their understanding of that material. (AM, Lesson Plan 3 Reflection, p. 3)

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[48] The term "reinforced" was used in a focus question for teacher's journal submissions. The question read: "How has what you are learning in Understanding Elementary Mathematics reinforced your beliefs about high quality mathematics lessons? changed them? In what ways? Give examples when possible." Therefore, teachers' use of the word reinforced was not spontaneous. The other terms were teacher-generated terms.

This focus on mathematical reasoning does not mean that teachers ignore computation, but rather that teachers have shifted their priorities when it comes to computation. Marie discussed how this changed for her in her second interview:

> Q: I want to read you this, your answer to one of these [quotes from your third journal] and just get your thoughts.
> MARIE: Okay.
> Q: So "as evident from the discussion above, UEM has impacted my opinions and beliefs about high quality math classes. Most profound is my new dedication to the idea that each student should be able to model and discuss the processes involved in solving a math challenge. In doing so, I believe they will grow as math learners and as lifelong problem solvers. Ultimately the ability to solve problems is far more important than math facts. In my second reflection, I spoke extensively about the impact of the mathematics practice standards on a high quality lesson. I still hold tight to those beliefs." I know you talked a lot about this already. Any other… what strikes you there?
> MARIE: Yeah. Something that you read struck me. This, this right here. Okay, so this is not something that I think I would have said before…
> Q: The math facts part?
> MARIE: Not necessarily. Yes. "So ultimately the ability to solve problems is far more important than math facts." So when they're modeling and arguing and discussing and forming their own ways of doing, or what they think at that time might be their own way; it might be shared with a bunch of others, but it's their way, they are getting themselves through the problem and, if we're going to talk about math being related to real life, which is another thing you probably would have read again and again, because I feel like that's a question kids always ask…
> Q: Yes.
> MARIE: To keep math, you know, important, we have to make sure it's connected to real life. But if we're really going to talk about that, then just the ability to get through a problem is so helpful in real life. So, we can attack a problem outside of school the same way as we do learn inside perseverance and all those things and I think that I would not normally say, that I would not normally let people off the hook about their math facts, 'cause you know it drives me crazy. [Laughs.]
> Q: Right, yeah.
> MARIE: But I believe there are more important steps where even if you don't know your math facts, you can still be included in this whole journey that

> [Laughs] we're taking through from beginning to end, through the problem, through the process of the problem.
> Q: Right.
> MARIE: I still want you to know what 4 x 8 is, though [Laughs]. (MG, Interview 2, p. 35-37)

Math facts are important to Marie, as they are to other teachers in the study; computation is still an essential part of math. But to these teachers, mathematical reasoning has also become a focus, and perhaps a more important one.

Seven of the eight teachers expressed the importance of student discussion of mathematical ideas in the learning of mathematics. Leona, who used a lot of discussion before the study, saw that her discussions with students became more focused on connections between mathematical ideas. Her example of the activity in which she asked her students to solve a problem using both multiplication and division[49] is one illustration of this. Only Sue did not specifically discuss the need for students to discuss their mathematical ideas with others; but Sue did recognize the importance of mathematical reasoning in her second interview when shared how she helped two students make sense of fractions by drawing out pictures of apples,[50] an example of making meaning in a way that Sue said she would not have done before her experience in the professional development.

More focused on inquiry. Teachers' belief statements revealed that they thought it was important for students to explore mathematical ideas and discover mathematical relationships. For example, instead of telling students mathematical procedures at the outset of a lesson, the teachers conveyed wanting students to develop understanding and explain procedures based on their understanding of mathematics. Marie

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[49] See also page 121.
[50] See page 76.

demonstrated this new focus on inquiry when she described what made her third lesson plan high quality:

> I think that this lesson is a high quality lesson because it offers students the opportunity to discover/uncover mathematical relationships before I write the procedure on the board! It helps them to become invested in the process by asking them to collect data, then apply their results without boundaries as to how that application should be done. The activity should promote discussion and sense making among the students. It should allow for multiple ideas/arguments to be made and understood. And promote reasoning, precision and modeling. (MG, Lesson Plan 3 Reflection, p. 1)

Bethan also shared how she has begun to allow her students to explore a topic in pairs before she gives them a "structured example" (BO, Interview 2, p. 11) on the board. Both Bethan and Marie expressed still believing that it is important to teach procedures, but said they would use student understandings to do so and would introduce procedures *after* exploration of the topic.

Alex, Marie, and Mary all discussed allocating time specifically so that students could explore and discover the mathematics. Ireane's shift from wanting to learn only one way to solve a problem to seeing the value in exploring multiple ways[51] is another example of this move toward inquiry. Leona explained why she believes inquiry and discovery is important:

> Letting them come up with the rules of math that I was taught as a kid will help them so much more because it has become their own discover; they own the skill at that point which makes the odds of them remembering and really internalizing that concept much greater than just giving them the rule and showing them how it works. (LS, Lesson Plan 3, p. 3)

More student-centered. Many teachers in the study exhibited a focus on students in their pre-training data. During the study, however, the nature of their focus

[51] See also page 98.

on students shifted. Although teachers began the study interested in individualizing instruction for students, they ended the study discussing the need to guide students on a mathematical journey. Ellyn discussed this idea as earlier than most, in her first journal entry,[52] but many teachers in the study expressed this idea by the end. Many teachers (Alex, Bethan, Ellyn, Leona, and Mary) used words like guiding or facilitating to express the ways in which they helped students develop their thinking about mathematics concepts. In her second interview, Mary said: “The teacher is more of a facilitator and a guide than the purveyor of all information” (MW, Interview 2, p. 29).

Teacher beliefs about the role of a math teacher. Taken together, the three shifts in teacher beliefs towards student-centered beliefs that are more focused on mathematical reasoning and inquiry provide a description of teachers’ perceived role in the mathematics classroom. Not all teachers experienced all of these shifts, but all teachers experienced some combination of these shifts. These changes led to a new definition of quality mathematics lessons and the teacher’s role in those lessons.

Leona’s description of a quality math lesson, from her second interview, is just one example of how teachers combine these three areas into one definition:

> Q: So a quality math lesson. What does it look like? How would you define it? What are its key characteristics?
> LEONA: Well, it’s having the students be able to come to the realizations of the understanding of the whys and hows and everything that we present as rules so much in the classrooms, but it’s taking the time back and letting them lead the discussions and the activities and come up with rules on their own. And then justify them and reason through them and apply them is, you know, such a huge thing. So that way they can kind of figure out, “Okay, this keeps working. So this is going to work.” And they’re doing most of it and the teacher is just kind of there, I think, to when they get those road blocks, to provide questions more to figure out, “Well how do we get around this?” and “What do you think

[52] See also page 146.

> if we did this?" And just kind of get their thoughts going again instead of, "Well, okay, let's sit down. This is how you do it." So no more of that. [I'm] Trying to get away from that. (LS, Interview 2, p. 15-16)

In Leona's description, the students are doing the work of discovering the mathematics. Leona's description not only includes all three shifts in teacher beliefs, but her description also defines the role of the teacher in a quality lesson. The teacher's role is to question and keep the students moving on their journey of discovery.

Most teachers' descriptions of quality are like Leona's; they combine beliefs about students, about inquiry, and about mathematics—and often discuss the teacher's role in this process. Mary's description of the changes in the ways she "looks at" her students is one example of this. Mary describes "looking at my students beyond their ability to compute and calculate … in order to guide my students as they discover their own understandings" (MW, Journal 3, p. 1).[53] It was difficult to find examples for the previous sections where the stated beliefs highlighted only one of the shifts in belief. To the teachers, these ideas are intertwined forming a vision of mathematics teaching that involves teachers guiding students as they discover mathematical understandings.

Less traditional, more constructivist. Two teachers in the study, Alex and Leona, began with relatively constructivist beliefs. They believed that knowledge of mathematics was constructed by students as they engaged in discussions and activities with mathematics. The other six teachers in the study moved along the traditional-constructivist continuum, landing somewhere more constructivist than where they began. The distance traveled along the continuum varied greatly. With no objective

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[53] See also page 84.

measure of distance on the continuum, I have chosen to describe each teacher's movement as large or small.

Ellyn, Marie, and Ireane all exhibited large shifts toward more constructivist beliefs. Ellyn's change from valuing students' answers to valuing their learning process is one strong example of her shift.[54] Ireane described a similar shift in her beliefs[55] from "teacher-directed" (IC, Interview 2, p. 16) to "investigative" (IC, Interview 2, p. 16). Marie's discussion of her change to "accounting for understanding" (MG, Interview 2, p. 3) and "the enjoyment of … figuring anything out for themselves" (MG, Interview 2, p. 3)[56] shows that same sort of movement along the continuum. These teachers changed their idea of what it means to learn mathematics. They used to think of mathematics knowledge as something that was given to a student. Now they think of mathematics knowledge as something the student discovers or builds.

In contrast, Bethan, Mary, and Sue have small shifts towards more constructivist beliefs. Bethan discussed allowing students to "explore" (BO, Interview 2, p. 12) with the mathematics and her greater appreciation of the multiple ways students approach problems and the value of discussion in the classroom. However, she makes other statements that indicate that she is still much more comfortable doing, and often falls back on, what she knows "will work" (BO, Interview 2, p. 2).[57]

Mary began the study speaking of students working together to build mastery of procedures. Although she described using manipulatives and examining student thinking, both aspects of teaching methods consistent with constructivist beliefs, Mary

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[54] See also page 106.
[55] See also page 99.
[56] See also page 114.
[57] See also pages 88-92.

used these procedures to reinforce set procedures rather than a construction of knowledge. Her view of learning was a traditional one. Mary describes moving to a view of mathematics that includes more than procedures; it also includes the "whys and wherefores" (MW, Journal 3, p. 1).[58] She is not confident in her ability to understand those whys and wherefores herself and this limits her movement towards methods more consistent with constructivist beliefs.

Before her experience in the professional development training, Sue saw mathematics as a series of steps. She discussed her role in student learning as finding the areas where the students were making mistakes, sharing her "thinking" (SD, Interview 1, p. 10) as she showed the manipulations of numbers or symbols, and the presentation and practice of one "rule" (SD, Interview 1, p. 10) at a time.[59] Sue continued to value the algorithmic nature of mathematics throughout the study. She did begin to appreciate the value of conceptual understanding as well, but she still discussed "teaching the concept" (SD, Interview 2, P. 3).[60] Her choice of words shows that she still sees knowledge as being transmitted from teacher to student, a traditional viewpoint.

Teachers' places on the traditional-constructivist spectrum at the end of the study correspond roughly to their years of experience in teaching. The longer they have been teaching, the more traditional their beliefs. I caution readers not to assume any causality, or even any true correlation, from this correspondence. This study provides too small a sample to be a correlational study. It is possible that there is a correlation between years of experience and position on the traditional-constructivist spectrum. It

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[58] See also page 84.
[59] See also page 73.
[60] See also page 76.

is just as possible that there is a correlation between exposure to learning mathematics through inquiry and age, and that this correspondence drives one's position on the traditional-constructivist spectrum. This study does not address either of these possibilities; I state them as a caution against reading too much into the data.

What can be stated from the data is that there does not appear to be a relationship between teachers' roles and their positions on the traditional-constructivist spectrum. Interestingly, among the three teachers with the largest shifts towards constructivist beliefs were one general education lead teacher (Marie), one special education lead teacher (Ellyn), and one mathematics support teacher (Ireane). Similarly, among the teachers with the smallest shift toward more constructivist beliefs were also one general education lead teacher (Sue), one special education lead teacher (Bethan), and one mathematics support teacher (Mary). Mathematics teaching role appeared to have no bearing on the amount of change in teachers' beliefs. Aside from the pattern according to teaching experience noted above, there appeared to be no pattern in teachers' during- and post-training beliefs according to demographic information collected.

Teacher beliefs about themselves as math teachers. This theme in the data came as a complete surprise. I expected to hear about teachers' beliefs about their own mathematics ability, but I expected that as teachers gained mathematics ability, their beliefs about their ability to teach mathematics would also improve. This was not the case. Teachers expressed various levels of confidence in their own ability to be a competent math teacher and those levels of confidence did not directly correlate to the level of content knowledge of the teacher.

In the pre-training phase, Ireane, Leona, and Mary expressed feeling the desire to learn more mathematics content in order to be more comfortable with their own mathematical abilities and teach children. Through the professional development, Ireane gained confidence in her ability to learn math, but felt she still had much more content to learn before she would feel confident teaching math. Leona expressed an increase in her confidence level as a result of her experience in the professional development and the new knowledge she gained. “I feel so much more confident in my reasoning,” she wrote in her third journal, “and I also feel better about admitting that I don’t know everything there is to know about math because I was able to work with many people who are in that same boat with me who teach math” (LS, Journal 3, p. 2).

Mary, on the other hand, said, “after the workshop I’m still really doubting, I think I’m even doubting more of my own abilities as a math teacher” (MW, Interview 2, p. 7). In her second interview she worried that she did not know enough mathematics to teach her students well:

> So, I guess I’m in a quandary. I feel very much like I almost need my hand held till I get more forward movement [in her math knowledge]. I don’t know. it’s something I’ve really been thinking about, and it makes me sad, because I don’t want them—as I think I said in my other interview, I don’t want them to feel about math the way I feel about math. But then am I making them feel about math the way I feel about math because of my lack of knowledge? I hope not. (MW, Interview 2, p. 11)

Later in the interview Mary addresses her doubts in her own ability again:

> But I think, as an educator, that’s what’s important, to understand the whys. Why does this work? Because that’s what kids want to know, especially when something gets tough, ‘Why do I need to know?’ It’s the why, it’s like they’re two. ‘Why? Why?’ And you’ve got to be able to address that in an intelligent manner. And maybe it’s not so much a doubt in my own ability, it’s a doubt,

> can I explain the whys, or the rationale behind [the math]. (MW, Interview 2, p. 18)

This is a struggle for Mary, balancing her belief that she may not be the best person to teach math to her students because of her low level of content knowledge, with her belief that she is trying to find ways to improve her own knowledge and help her students to the best of her ability.

Ellyn also expressed lower levels of confidence in her ability to teach math after the professional development training. Before the professional development training she had been considering working towards earning highly qualified teacher[61] (HQT) status in math. After the training she was reconsidering that decision. She volunteered this information in her second interview:

> ELLYN: The sad thing is, and you'll be really sad to hear this. I had been considering getting, taking a crack at to be HQT in teaching math. I don't think I'm going to do it.
> Q: How come?
> ELLYN: I don't think I'm the greatest math teacher. I think I'm better off in the role that I'm in, which is a special education teacher. … Thinking about the teaching of the math and then sometimes I just, I, I'm not sure that I'm—maybe it's 'cause I feel coming out of this is that I need to learn how to be a better math teacher. That's one thing I've learned from this. I thought I was an okay math teacher. I thought I was okay. But now, I'm like, I learned, I almost learned like how much I don't know. (ED, Interview 2, p. 25-26)

Interestingly, during the cross-case participant verification Ellyn commented on her post-beliefs statements about her confidence level. "It's funny that I felt that I wasn't necessarily a good math teacher at the end of the study. I feel much more confident this

[61] Highly qualified teacher status is a designation under the No Child Left Behind Act of 2001 that says all teachers must have a certain level of knowledge in the content areas that they teach. A teacher who has been granted this status by their state's Board of Education is called HQT.

year" (ED, Cross-Case Verification, p. 5) This statement indicates that changes in levels of confidence may be temporary rather than long-term.

This idea of "learning how much I don't know" (ED, Interview 2, p. 26) had different effects on teachers' confidence in their ability to teach mathematics. Mary and Ellyn experienced declines in their confidence levels, others did not. Ellyn expressed that her lower confidence was temporary and that she may have actually experienced an increase in confidence. Alex, despite learning how much she did not know, did not question that she would continue to explore her own mathematical understandings. She expressed feeling like others would benefit from the same training she had received and that she would like to do another, similar training on higher level content. Alex wanted more training, but did not feel that this made her less qualified to teach mathematics. These differences in confidence are intriguing and point to the need for future study on the relationship between training in mathematics content and self-efficacy.

The special case of Sue. Sue Daniels is a bit of a puzzle. She was very different than other teachers in the study, both in her teaching circumstances, her participation in the professional development, and her changes in content knowledge and beliefs. These differences are worth examining.

Recall that Sue teaches in an online environment. She does not teach face-to-face, although she does interact one-on-one with her students via technology. In addition, Sue's lessons are provided to her students directly via the computer. Sue does not control those lessons. Rather, Sue controls the aspects of the learning that flow from those lessons. She provides help when students get stuck, redirection when students are off-track, and assessment of student learning. Sue's lessons do not and cannot, due to their online and prescribed nature, involve much student interaction. Other teachers in the study discussed increased interaction between students through discussion or problem-solving tasks. Sue did not. This may be because Sue did not see how they could fit into the set structure of her prescribed lessons where individuals take her course independent of other individuals, not because she did not see the value of these aspects of the professional development. Sue does not have a physical classroom, and so the nature of classroom interactions must be different.

Sue had relatively strong content knowledge at the beginning of the study, according to her MKT scores, but saw a measured decline after the professional development. Mary also had a decline in scores, but had a technical issue with the administration of the test that may have impacted her scores. Sue did not have technical issues; neither the validity nor reliability of Sue's scores was threatened. Sue's data does not provide an explanation for the decline.

Sue's beliefs about quality lessons were beginning to change by the end of the study. However, the extent of those changes was much smaller than the changes in other teachers' beliefs. Sue missed approximately one-third of the professional development training, more than any other teacher. This provides one possible explanation for why Sue's changes in beliefs are less developed than other teachers in the study. Sue may have been on the same journey as other teachers, but was not as far along due to her smaller amount of time spent in class experiencing learning differently. This smaller amount of class time, coupled with the differences between the interactive nature of the learning environment in the professional development and

Sue's online teaching environment, may have impacted her ability to transfer her experience in the professional development to her beliefs about the ways in which learning could occur with her students.

Teacher learning. Although I did not set out to study teacher learning, the teachers in the study shared what they learned, how they learned, and how that learning impacted them. Teachers shared how the very act of learning mathematics changed their perceptions of mathematics, their beliefs about quality lessons, their ways of working with students, and their ways of thinking about themselves. Without examining these statements in the data I would not have fully understood teacher's changes in beliefs about quality lessons.

The learning of mathematics. The third initial research question in this study is "How do the statements, goals, and strategies change as the teachers gain mathematical content knowledge?" This question relies on the assumption that the teachers did gain content knowledge. Teachers' content knowledge was examined in two ways: via MKT scores, and via qualitative data from teacher submissions and interviews. Although the MKT data does not show teacher growth in content knowledge, teacher statements in the data support do that the teachers gained content knowledge, particularly in the areas of conceptual understanding of mathematical procedures and connections among mathematical concepts.

MKT data. After my initial analysis of the qualitative data and the completion of the within-case verification, I analyzed the data from the pre- and post-training administration of the MKT measures. This pre- and post-test data was collected in order to verify that the teachers did gain content knowledge. Both forms[62] used have a standard error of measurement of 0.43175 (H. Hill. personal communication, November 19, 2013). I created error bars for teachers' scores and examined individual differences. It is important to note that this use is outside the intended use of the measures. Though the MKT measures were designed to compare groups of teachers, not to make "highly accurate statements about individuals' mathematical knowledge" (Study of Instructional Improvement, n.d.), the standard error of measurement does make it possible to examine individual teachers' knowledge. Each teacher's change in content knowledge individually was discussed as part of each teacher's within-case analysis (see Figure 6 through Figure 13). The discussion in this section will focus on what these individual's scores say about the group as a whole.

[62] The forms used are the elementary number concepts and operations forms, 2008.

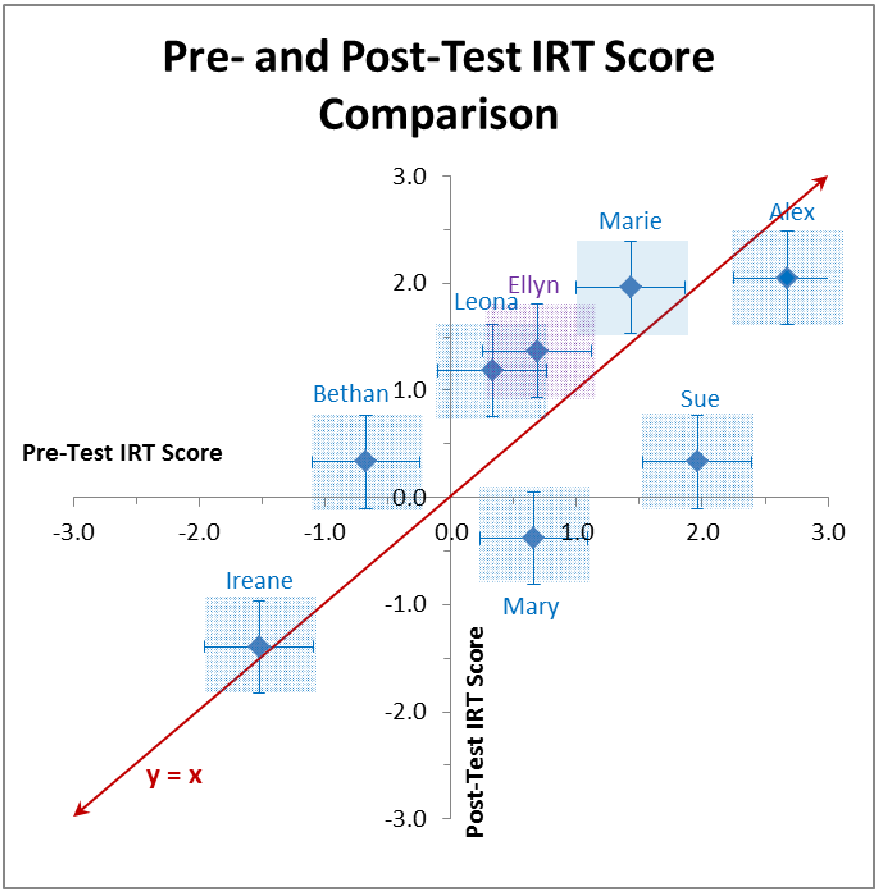

Figure 15. Teachers' pre- and post-training IRT scores on the MKT measures. Error bars represent the standard error of measurement of 0.43175. Rectangles are formed from the error bars to make it easier to see the limits of the error. Color is used to differentiate teacher data; color does not have meaning in and of itself. The y=x line is added to show equivalent pre- and post-training IRT scores.

The scatter plot in Figure 15 is useful for examining trends in the data. Three teachers' pre-and post-training score rectangles in Figure 15 do not intersect the y=x line. This indicates that only Sue, Mary, and Bethan had pre- and post-training scores where there is more than a 68% chance that their true scores are distinct from one another. Sue and Mary scored lower on the post-training administration than on the pre-training test. Bethan scored higher on her post-training test than the pre-training test. This can be seen by the teachers' rectangles location below or above the y=x line, respectively. Although most teachers' scores were within the standard error of measurement of the test, as indicated by the rectangle's intersection with the y=x line, the scatter plot indicates that there may be a trend in the direction of higher post-training scores. The group size of this sample and the reliability of the MKT measures are too small to draw any conclusions about this potential trend.[63]

In their second interviews, before analyzing the MKT data, I asked teachers how they felt about their post-training MKT experience. I wanted to know how teachers felt they did and if they felt differently about the post-training assessment than the pre-training one. In general, the teachers had no strong feelings either way about either of their performances on the MKT assessment. Two comments stuck out. Ireane said that she felt like her pre-training assessment was testing facts whereas her post-training assessment was testing the process and not whether or not she could solve the problems. Sue said that she felt like the pre-training assessment tested her math abilities and the post-training assessment tested how she would teach those topics. Although it appears that Ireane and Sue agree on the content of the pre- and post-training assessments, Ireane and Sue actually took opposite forms of the test! Ireane took Form A as her pre-training assessment and Form B as her post-training assessment while Sue took Form B for her pre-training assessment and Form A for her post-training assessment. This indicates that these two teachers may have changed the

[63] The developers of the MKT measures suggest a group size of 60 teachers in order to make comparisons over time (Study of Instructional Improvement, n.d.).

ways in which they think about mathematics problems, not that the assessments measured different constructs.

Qualitative data. The teachers in this study unequivocally state that they learned mathematics during UEM. Each teacher was able to give at least one, and usually many, specific examples of mathematics content that they felt they had learned through their work in the professional development. Leona describes how she changed her beliefs about what mathematics is as a result of learning mathematics.

> Overall, I feel like I have a better understanding of a lot of different ideas than I had before and just being able to see and express it different in different ways has helped me change my thinking about what math is. It's not just one set of rules that we have to keep following, but it really is related. If you can do it with whole numbers, then you can probably do it with fractions, you can do it with decimals. And it's finding those patterns and relationships that I was slowly getting, but this – it was good because I was two months worth of okay, jumpstart. Let's get this going. (LS, Interview 2, p. 19)

Other teachers in the study (Alex, Ireane, and Marie) also discussed learning about the connections among mathematical ideas.

Many teachers felt that they had gained knowledge of particular aspects of fractions. Bethan talked about her understanding of fractions and thinking about "a part of a part" (BO, Interview 2, p. 1). Leona discussed understanding the reasons behind the procedures for operations with fractions. Mary also discussed learning the rationale behind the procedures but expressed an uncertainty in her ability to explain the rationale to others. "I would have to do it again and again and again in order to have in more ingrained" (MW, Interview 2, p. 19), she said. Marie learned about the area model for multiplication and how useful it is for multiplying fractions. Alex described feeling as though she had a complete understanding of division of fractions before she began UEM but then gained a "more nuanced" (AM, Interview 2, p. 14) understanding of division of fractions. "I guess if you were to visualize my understanding of division [of fractions]," she said in her final interview, "it went from being this very Charlie Brown Christmas tree looking thing into a much bigger, more filled out, more elaborately decorated Nutcracker type of Christmas tree" (AM, Interview 2, p. 14).

Other teachers in the study also described a deepening of their understanding of the mathematics. Ellyn described the idea of viewing subtraction as the difference between points on a number line. Sue learned why subtracting a negative is adding a positive. Ireane shared that during the post-test she felt that for the first time she was able to understand enough of the mathematics to know what the questions were asking.

> Reading the posttest questions, in particular the word problems, my knee jerk response to turn away and run did not kick in. Why? I wondered.… For the first time I actually understood what question was being asked. How empowering that feels to be able to read a word problem and somehow conceptualize what's taking place. (IC, Journal 3, p. 1)

In addition to naming particular topics that they learned or learned more about, the teachers talked and wrote about learning how others viewed various topics and problems. Without exception, teachers talked about how useful it was for them to see the many different ways they were able to solve the same problem in the professional development and how this helped them appreciate students' problem solving methods. This knowledge of other ways to solve a problem is one important component of mathematical content knowledge.

Reconciling the data on teacher learning of mathematics. At first glance it appears as though the quantitative and qualitative data on teacher learning of mathematics is contradictory. Teachers stated they learned mathematics, but the quantitative data from the MKT assessments did not show statistically significant increases in teachers' IRT scores. There are several possible explanations for this disconnect. The MKT assessments may not have had enough questions about the specific mathematical knowledge that teachers gained and hence teachers' scores did not change enough to be outside the standard error of measurement. The Study for Instructional Improvement states that the MKT is best suited for use with "programs that intend to broadly improve teachers' knowledge in any of the content areas named above [which include elementary number and operations]; programs which focus only on narrow bands of the K-8 math curriculum (e.g., programs which focus only on linear functions) will not likely see positive results" (Study of Instructional Improvement, n.d.). Although the professional development used in this study did provide instruction across many areas of elementary number and operations concepts, teachers began the study with knowledge in those areas. Teachers may have improved in specific content areas related to number and operations concepts, but may not have had broad improvement across all areas tested. Those areas where teachers improved may only have appeared one or two questions on the assessment, resulting in very little change in overall score.

Recall also that the teachers' IRT scores on the MKT assessments (see Figure 15) indicated that there may be a trend in the direction of higher post-training scores. Two of the three teachers whose post-training MKT scores were lower than their pre-training scores had circumstances that make their scores suspect. Mary's post-training score was earned after she had technical difficulties with the testing program.[64] Alex's scores are high enough that her scores may have been impacted by the MKT

[64] See page 81.

assessment's known ceiling effect. Conducting the study again with a larger sample would allow for a group size large enough where it would be possible to see statistically significant changes in knowledge as measured by the MKT assessment, should such changes exist, despite such testing issues. With such a small sample size the apparent incongruence between the quantitative and qualitative data is not entirely unexpected.

The learning of pedagogy. Far more important to teachers than the mathematics content they were learning, was how they were learning it. Many teachers in the study already knew methods for finding answers to the mathematical questions posed in the professional development training. The teachers in this study described learning about how to teach mathematics by observing the ways in which the professional development was taught and thinking about their experience learning in these ways. Experiencing learning in this way impacted them more profoundly than the learning of content.

Teachers often referred to their observations of the ways in which the professional development was taught as modeling. I chose not to use this term for two reasons. (1) The term modeling is often used in mathematics to mean the use of mathematical models. Mathematical models are ways of visualizing, demonstrating, or thinking about mathematical constructs. The area model of multiplication, for example, is one mathematical model. Additionally, the term mathematical modeling is used to describe the use of mathematical tools to analyze or make predictions about real world situations or phenomena. For example, mathematical equations can be used to describe or model the Earth's orbit around the sun. Using the term modeling for demonstrating mathematical learning and for mathematical modeling invites confusion. (2) The term modeling implies intent on the part of the instructor to demonstrate a particular method and for the teacher to also interpret it as such. Although the instructor of the professional development did intend to use best practices for mathematics teaching, she did not do so for the purpose of demonstrating methods but rather to further teacher learning. I decided to call this phenomenon *learning to teach through participation in learning*.[65]

Teachers described the value of the professional development for helping them see first-hand the struggle of solving problems, the many different and correct ways that problems could be solved, the reasoning behind procedures, and the connections between mathematical ideas. They talked about persevering and experiencing the standards for mathematical practice.[66] The teachers continually connected their experience as a learner to their classrooms. It was rare for a teacher to talk about her own experience without also reflecting on what that experience must be like for her students. Ellyn described the importance that learning mathematics had in changing her teaching:

> Being a math student was very informative because I had to learn things I wasn't sure how to do. I had to think about how I was processing it. More instructional for me than anything, I think, was realizing how different all of us in that class did it; and that we still would all come to the same answer, if the answer was what we were seeking, but in such a variety of ways. Even all the ones [different algorithms used to solve computation problems] that she [the instructor] put up on the wall that we had to think about, we had to go through

[65] See also the discussion of Lortie's (1975/2002) apprenticeship of observation as discussed in Chapter V.

[66] The standards for mathematical practice are part of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These standards deal with the act of doing mathematics rather than procedural and conceptual knowledge to be gained.

> and, and think about what was being done without being told what we were looking for. That was an interesting way to go through and realize that people are doing it all differently and that's perfectly fine. The goal is to find out what allows someone to come to a solution and to figure out [that] if they do it in a way that's different than I do it, or if they don't do it the way this step said in the book, that's fine—if they can articulate what they're doing and it makes sense. So, I've been interested to watch my students in a different way as a result of what I went through as a teacher, as a student of math. (ED, Interview 2, p. 2-3)

Bethan described what the experience of learning mathematics through inquiry was like for her:

> BETHAN: We did a lot in that course! But you know until you really think about it and put it into words… I mean some of those nights were painful.
> Q: Tell me more.
> BETHAN: Because I just didn't like look at [a problem] and display what I'm thinking. It took me awhile to warm up to that you know. But once you did and you realize that everyone was doing it in their own way anyway and there was really no one right way because you were all exploring what to do with it and present it. And I think that was challenging, some nights painful, but in the long run I think it was positive. At least I view it as positive.
> Q: Yeah, why?
> BETHAN: Because I think we all need to expand our horizons and look at how we can arrive at the same place on a different track so that we all can be self validated with what we do, I guess. And to know inside we can bring forth what needs to be done and certainly that's what you want to teach kids: is to take the risk, make a mistake, learn from the mistake and go forward. And share what you do know but share your mistakes, too, because others can learn from your mistake. So it's a sense of community, I guess, and feeling comfortable enough to be challenged within it. And you want the kids to take a risk, feel comfortable, and yet be challenged.
> Q: Sure. That's pretty cool.
> BETHAN: Yeah it was. But some nights it was still painful. [Laugh.] (BO, Interview 2, p. 12-13)

Just as Ellyn drew a connection between her learning and that of her students, Bethan made the connection between the ways in which she was asked to interact in the classroom and the ways in which she would want students to interact in her classroom.

This same theme, that the act of learning mathematics through inquiry impacted teachers' thoughts about the teaching of mathematics, appears, in some form or another, in statements from nearly every participant in the study.

Teachers learned how to use inquiry as part of their mathematics instruction. Alex discussed learning how to use problems that are simple in structure to help students discover mathematical structure. Bethan shared having the students provide examples for problems before she shared a structured example. Leona learned how to ask better questions to elicit student thinking. Ireane and Mary discussed having their students share the multiple ways they solved problems. Marie, in her second interview said:

> I think I have this ideal situation in my head now that says, Okay, each new thing, each new concept should begin with this: "Here's this stuff and what can you do with it?" And then after they have a chance to try their thoughts and procedures and whatever and then share them and then see if they're like the other people's, because now that I've been doing that, I'm kind of amazed at what my kids are coming up with. (MG, Interview 2, p. 4-5)

Not only did teachers learn how to teach using inquiry, they credit their experience learning in this way with teaching them this pedagogical strategy. In her third journal, Bethan wrote:

> I think that this class has widened my belief of what makes a quality lesson. I believe that this class has made me think more about different ways one concept can be looked at and evaluated. It has made me aware that discussion, decomposition and sharing has to be a much larger part of the lesson. (BO, Journal 3, p. 1)

Ellyn discussed, in her third journal how she learned to teach using inquiry through watching the instructor: "[The instructor] is an excellent teacher and role model of how to teach through student exploration. I have learned just by watching her" (ED, Journal 3, p. 2). In her second interview, Ellyn elaborated:

> She was a great role model for me and I felt that the way she taught it gave me someone to watch who had a really good way of going about getting people to think for themselves and not giving them answers. So, that was really, really positive role modeling. So she was constantly making me think about how I taught and what I was taking away from each class to school the next week. (ED, Interview 2, p. 31)

In her third journal, Mary wrote about the ways in which the participating in the professional development impacted her pedagogical skills.

> Scaffolding throughout the "learning ladder" provides the continuum between concrete, representational, and abstract approaches to concepts. Scaffolding needs to jump back and forth between different levels of understanding in order to monitor upwards progression in understanding. By participating in this course, I was better able to monitor my own metacognitive movement between those "steps" of understanding. (MW, Journal 3, p. 1)

Marie and Sue both talked about learning that just because a student has the right answer, it does not necessarily mean that they understand the mathematics. "I definitely know that by just learning the rules and the traditional algorithm, which is also what I really mean by the rules, is zero guarantee of understanding. Or relatively zero" (MG, Interview 2, p. 35), Marie said in her second interview. "It's a guarantee of you can memorize. And follow procedures" (MG, Interview 2, p. 35). Sue described the same realization in her second interview:

> and of course we had had those questions in the pretest and I was like, "Who ever thought of this?" … So I could see how the kids were working through it, because I'm really good at picking out what they did wrong and helping them correct their work. But if they get it right, I don't always see that they're not really understanding it. And I got that part. And so that was another piece that I personally learned, that just because you've got the right answer doesn't mean you understand it. (SD, Interview 2, p. 28)

Marie and Sue both felt that they had gained a new pedagogical skill because they were now able to recognize the need to examine students' understanding of the mathematics in ways above and beyond examining the students' answers.

These teachers gained pedagogical knowledge; they learned to teach mathematics as a direct result of their participation in the learning of mathematics, and recognized that their learning experience gave them these skills.

Inquiry-based learning as a driver of changes in knowledge and beliefs. I have discussed teachers' changes in beliefs, in knowledge of mathematics, and in knowledge of pedagogy. As discussed above, in all of these instances, teachers credited, often directly, their changes in content knowledge, changes in pedagogical knowledge and changes in belief to their experiences as a learner in the Understanding Elementary Mathematics professional development and to the inquiry-based methods of instruction used within the training.

Recall that the third initial research question in this study was: How do the statements, goals, and strategies change as the teachers gain mathematics content knowledge? Recall also that teachers' goals and strategies were often invisible in teachers' lesson plans. This, coupled with this new finding that it was the act of learning through inquiry that changed teachers' content knowledge, knowledge of pedagogy, and beliefs about quality lessons, makes the third research question irrelevant. It is not gains in content knowledge that changed these teachers' beliefs about quality lessons. Rather, it is the act of experiential learning that changed teachers' content knowledge, pedagogical knowledge, and beliefs about quality lessons. This research question was revised to become: How does the experience of learning mathematics content through inquiry change teachers' beliefs about what constitutes a "quality mathematics lesson?"

Summary of cross-case findings. A cross-case analysis of the data from all eight teachers yielded findings in two main areas: teacher beliefs, and teacher learning. First, the data showed that teachers' beliefs about quality lessons became more focused on mathematical reasoning, more focused on inquiry, and more student-centered. In addition, teachers began to see their role as that of a facilitator, guiding students on a journey of inquiry to gain mathematical understandings. Second, the data showed that teachers learned as a result of their experience as students. The teachers in this study learned both mathematics and pedagogy, even though only mathematics content was specifically taught. Further, teachers directly credit the act of learning as the leading factor influencing these changes in their knowledge and their beliefs.

Conclusion

This multiple-case study examined eight teachers' beliefs about quality mathematics lessons. Each of the eight teachers in the study was considered as a single case. Data was analyzed within- and across-case in order to examine the data for the richness of individual descriptions of the learning process and the themes expressed by the multiple teachers. While examining these themes the research questions were adapted to better align with the data. The research questions for the study became:

- What do experienced K-8 teachers believe constitutes a "quality mathematics lesson?"
- How does the experience of learning mathematics content through inquiry change teachers' beliefs about what constitutes a "quality mathematics lesson?"

The cross-case analysis uncovered findings in two areas: teacher beliefs about quality lessons, and teacher learning. These findings will be discussed in the context of the research questions in the next chapter.

Chapter V: Discussion of Findings

This study produced findings in three main areas: (1) teachers' pre-training beliefs about quality mathematics lessons; (2) changes in teachers' beliefs; and (3) teacher learning. The first section of this chapter describes teachers' beliefs about quality mathematics lessons at the beginning of the study, before training in mathematics content. The second section of this chapter discusses changes in teachers' beliefs and describes teachers' new vision for professional practice that incorporates these changes in belief. The third section argues that changes in teachers' beliefs, knowledge of content, and knowledge of pedagogy were all a function of their experience as a learner of mathematics.

Teachers' Pre-training Beliefs about Quality Mathematics Lessons

Teachers' pre-training data and belief statements about quality mathematics lessons created a description of lessons in which teachers provide a safe environment in which students can learn, and in which students are actively engaged in their learning. Five of the eight teachers also discussed the importance of providing step-by-step procedures for students to follow and half of the teachers discussed the importance of individualizing instruction for students. Only two teachers emphasized students discussing mathematics and only one teacher's statements involved having students develop and discover the mathematical ideas behind tasks.

Although all of the teachers in the study discussed engaging students in the mathematics, the definitions of six of the eight teachers described superficial engagement in tasks that involved student activity, not student reasoning and meaning-making. The teachers described promoting student engagement through the use of activities that involved whole-body movement, were fun, connected to real-life topics such as money or food, or engaged students in the task of figuring out where they made mistakes. The teachers' definitions are in contrast to their own post-training beliefs and the meaning of cognitive engagement outlined as the *student participation in reasoning and meaning-making* dimension of MQI.

Two teachers, Leona and Alex, did describe components of quality lessons that matched the MQI dimension of student participation in meaning-making and reasoning, but to differing extents. Leona discussed classroom discussions about mathematics topics where students explained mathematics and why a strategy would or would not work. After participating in the professional development, Leona said that her discussions were very different than they were before; she described asking questions that forced her students to "figure out the patterns on their own … [and make] connections" (LS, Interview 2, p.1). This shows that even though Leona required students to make meaning and reason about mathematics before the professional development, she did not do this in a high quality way. Alex discussed students reasoning about and making meaning regarding mathematical topics before and after the professional development. Her descriptions involved students building the ability to reason, discuss, and problem solve. These descriptions do represent high quality instruction according to the MQI dimension of student participation in reasoning and meaning-making.

Although only two teachers expressed student engagement as reasoning and meaning-making, all teachers discussed engagement of some kind. Without close examination of the teachers' beliefs, it would have been easy to incorrectly assume that when the teachers discussed engagement they meant the same things as published descriptions of quality. Ross et al. (2002) write that

> the most important obstacle [to substantial implementation of reform teaching] is that teachers' beliefs and prior experiences of mathematics and mathematics teaching are not congruent with the assumptions of the Standards. Teachers mostly support the goals of reform, but overestimate the extent to which their practices approach these goals. (p. 132)

This overestimation is apparent in the teachers in this study. They believed, before training, that they were engaging students in mathematics. After training, they believed that engagement required student engagement in mathematical reasoning. Teachers' belief statements indicate that, had they been asked a survey question about the importance of student engagement before participation in the professional development training, they likely would have answered that they believed it was important. These answers would have been misleading due to differing definitions of engagement. These findings indicate that researchers must be wary when examining teacher beliefs and when interpreting data about teacher beliefs.

In one of the few studies examining teachers' beliefs about quality lessons, Wilson et al. (2005) asked experienced mentor teachers to describe successful lessons that they and their student teachers had implemented. In contrast to this study, the teachers in Wilson et al.'s study were never asked directly what they believe "constitutes good mathematics teaching" (p. 90). The teachers in that study believed that "good teaching requires a sound knowledge of mathematics, promotes mathematical understanding, engages and motivates students, and requires effective management skills" (p. 83). The teachers in this study agreed with the first three requirements from the Wilson et al. study, although the agreement was not always explicit. The teachers in this study expressed the importance of content knowledge throughout their statements, but did not list content knowledge as a characteristic of quality lessons, perhaps because that is a characteristic of the teacher rather than the lesson. The teachers did recognize the importance of content knowledge on their teaching. Four teachers (Ellyn, Ireane, Leona, and Mary) specifically expressed feeling, either before or after the professional development, as though their lack of content knowledge limited their ability to provide high quality instruction. The teachers in this study often expressed the need to promote mathematical understanding and engage students although, as discussed above, their meaning of these terms did not necessarily match the literature's meaning of these terms. Wilson et al. do not discuss what the teachers in their study meant by these terms and so it is not possible to determine if the teachers in that study shared similar, superficial meanings of the terms understanding and engagement.

The last requirement from the Wilson et al. (2005) study, effective management skills, was absent from this study's data. This may be due to the different population and circumstances of this study and the Wilson et al. study. In the Wilson et al. study, the researchers asked experienced teachers to describe quality lessons implemented by themselves and by their student teachers. Student teachers typically have not yet developed classroom management techniques (Hollingsworth, 1989; Putman, 2009); the experienced teachers in this study have. The teachers in this study may also believe that classroom management is needed, but their descriptions of quality focus on classroom climate and the need for a safe learning environments, not simply managed ones.

Changes in Beliefs about Quality Mathematics Lessons

Teachers experienced three main shifts in their beliefs about quality lessons; their beliefs became more focused on mathematical reasoning, more focused on inquiry, and more student-centered. While searching in the literature for ways to understand my findings related to teacher learning, I stumbled across *Preparing Teachers for a Changing World* edited by Darling-Hammond and Bransford (2005). This volume compiles the research on teacher learning with a focus on the preparation of new teachers. In the introduction, Bransford, Darling-Hammond, and LePage (2005) present a framework for understanding teaching and learning (see Figure 2). Two of the circles in this UTL framework represent knowledge of subject matter and knowledge of teaching. These overlapping circles in the image matched the relationship between content and pedagogy in Ball's theoretical framework of mathematical knowledge for teaching (as in Ball et al., 2008). The third circle in the UTL framework represented knowledge of learners. I was reminded of my data regarding classroom climate and began to examine if and how the UTL framework fit my data.

The three areas of deepening of beliefs described by teachers – more focused on mathematical reasoning, more focused on inquiry, and more student-centered – correspond to the circles in the framework for understanding teaching and learning. If we consider the framework for understanding teaching and learning also as a framework for understanding teacher beliefs about mathematics teaching and learning, these beliefs can be mapped to this framework (see Figure 16). Because of the exact match between these circles and the three areas of teacher beliefs outlined by NCTM, it makes sense to do this. Rather than map every change in belief, I have chosen to examine three changes in beliefs in detail, one from each major shift in belief. Teacher beliefs became:

- more focused on mathematical reasoning, as occurred with teachers' changing definitions of engagement;
- more focused on inquiry, as in teachers' shift to providing time for exploration of student ideas before teaching algorithms;
- and more student-centered as evidenced by teachers' changing beliefs about their role as guide rather than provider of knowledge.

These three changes in belief are mapped to Darling-Hammond and Bransford's (2005) framework for understanding teaching and learning in Figure 16.

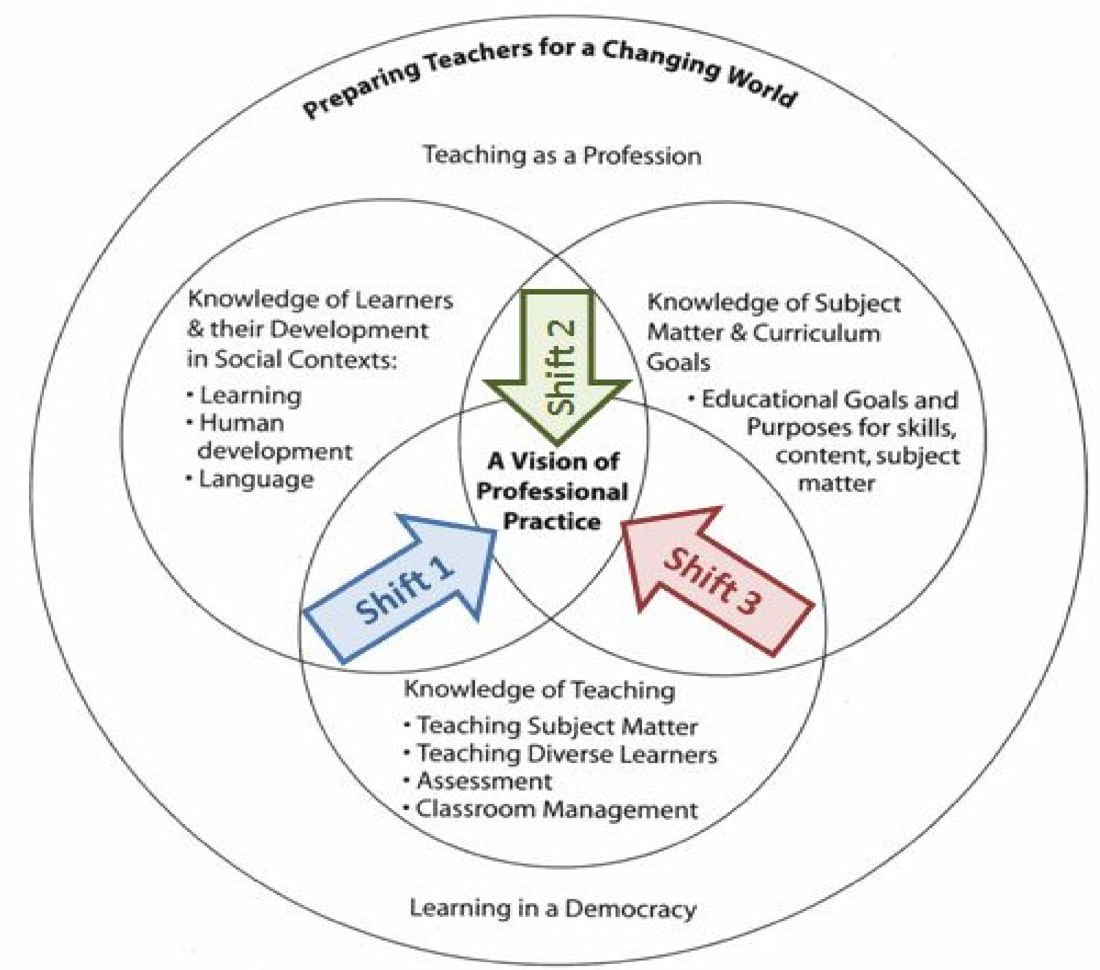

Figure 16. A mapping of teachers' shifts in beliefs onto the framework for understanding teaching and learning. Shift 1 represents shifts towards beliefs that are more focused on mathematical reasoning. Shift 2 represents shifts towards beliefs that are more focused on inquiry. Shift 3 represents shifts towards beliefs that are more student-centered. Adapted from *Preparing Teachers for a Changing World* (p. 11), L. Darling-Hammond and J Bransford (Eds.), 2005, San Francisco: Jossey-Bass. Copyright 2005 by John Wiley & Sons, Inc. Adapted with permission.

The first major shift in belief is towards beliefs that are more focused on mathematical reasoning. Teachers' changing definition of engagement is one example of this shift. Initially teachers' meaning of engagement sat at the intersection between the circles for teaching and learners. Teachers wanted to engage the learners in lessons that were interesting or relevant to them as human beings. By the end of the study, teachers believed that these learners must be taught using interesting or relevant tasks that engaged them in *mathematics*. This newly refined belief belongs in the intersection of all three circles (see Shift 1 in Figure 16).

The second major change in belief is a shift towards beliefs that were more focused on inquiry. The example used to demonstrate this shift is the move toward providing time for students to explore mathematical concepts before teaching algorithms with the intention of developing student conceptual understanding. Teachers described pedagogical decisions to allow time and activities that allowed students to explore and make their own meaning from the mathematical tasks in which they were engaged. This shift incorporates beliefs about teaching into teachers beliefs about how students work with mathematics (see Shift 2 in Figure 16).

The third shift in belief is a shift towards more student-centered beliefs. Teachers' beliefs about their role as guide is an example of this shift. This shift also demonstrates movement from an intersection of two circles to the intersection of all three. Many of the teachers initially believed that they needed to provide mathematics instruction, a belief about how mathematics is taught. When this belief about their role expanded to include guiding students on a mathematical journey, the third circle of learners was incorporated (see Shift 3 in Figure 16).

These three examples demonstrate how teachers' beliefs about quality mathematics lesson changed in ways that incorporated mathematics, learners, and teaching pedagogy together and moved towards the vision for professional practice described by Darling-Hammond and Bransford (2005) in their framework for understanding teaching and learning.

Similar to incorporating beliefs about mathematics, learners, and pedagogy, the teachers incorporated the dimensions of MQI into one another. Teachers' beliefs about lessons became more aligned with definitions from the literature of high quality lessons, but their beliefs changed in ways that went beyond MQI. Their shifts in beliefs correspond closely with the *richness of the mathematics*, *student participation in meaning-making and reasoning*, and *classroom work is connected to mathematics* dimensions of MQI, although none of the shifts map directly to any single dimension of MQI. Note also that the dimensions of *working with students and mathematics* and *errors and imprecision* were not visible in the data. Although teachers' beliefs about quality lessons did become more aligned with this definition of quality from the literature, MQI was less useful as a model by which to discuss the changes in teachers' beliefs than the framework for understanding teaching and learning.

When I began this study I expected that teachers' beliefs may focus more on mathematical content when they learned mathematics, and thus align better with definitions of quality from the literature. What I did not anticipate is the ways in which teachers' experiences learning mathematics would also make their beliefs more focused on learners and more focused on teaching strategies. What I never thought to anticipate is that teachers would develop a new vision of professional practice that would incorporate beliefs about mathematics, pedagogy, and students in defining the role of a mathematics teacher.

Teachers' new vision of professional practice.

I facilitate; [I] don't instruct. There's a big difference

between facilitating and instructing.

– Bethan, Interview 2, p. 12.

The teachers in this study developed a new vision of professional practice from their experience learning mathematics. This new vision centers on the idea that teachers guide or facilitate student learning of mathematics and that this is very different than providing knowledge or instructing students in methods that could be used to come up with answers to mathematics problems.

In examining teachers' statements from the beginning of the study, I found it was relatively easy to categorize teachers' statements into the three circles of mathematics, pedagogy, and students. In examining their later statements, I was having trouble deciding where their statements belonged. This was not because the statements did not fit in a category, but rather they incorporated all of the categories. Teachers' beliefs about quality lessons had deepened, and centered in on a new vision of professional practice. They saw their role in the classroom as a guide, guiding students as they discover mathematical understandings. Figure 17 provides a visual of this centering process.

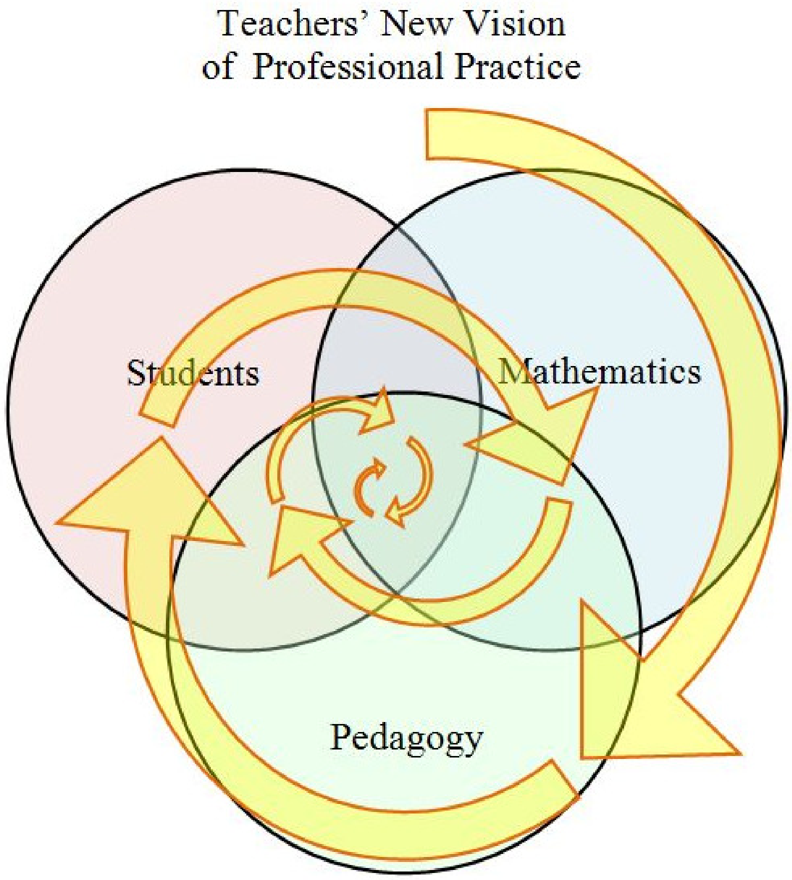

Figure 17. Teachers' statements of quality centered on a new vision of professional practice.

This study asked the research question: What do experienced K-8 teachers believe constitutes a "quality mathematics lesson?" Teachers in this study described their vision of quality mathematics lessons, and their role in creating them, as follows: *Teachers guide students as they engage in discovering and discussing mathematics and connections among mathematical content, building conceptual understanding, and practicing procedures. Teachers create and maintain a learning environment that allows students to engage safely and fully in the learning process. Teachers use* *multiple strategies, including strategies of inquiry, while guiding students. Those strategies may differ depending on the student, but all of the strategies help lead the student to discover and reason about mathematics in addition to developing strategies for computation.* This new vision incorporates many of teachers' initial views about lesson quality, but has become more integrated, more focused on mathematical reasoning, more focused on inquiry, and more student-centered.

A position for these findings in the research literature. Improving mathematics instruction may involve not so much a change in teachers' beliefs, Leatham (2006) argues, but a shifting of the relative importance of beliefs so that "'desirable' beliefs are seen by teachers as the most sensible beliefs with which to cohere" (p. 100). The teachers in this study engendered a new vision of professional practice that involved a shifting of the relative importance of their beliefs. This shift in relative importance can be seen in the teachers' beliefs about practicing procedures and about engaging with mathematics, a shift towards mathematical reasoning and inquiry. In teachers' pre-beliefs statements, they discussed the importance of students practicing procedures. In the during- and post-training data teachers stated that, although they still felt that procedures were important, they believed that students needed to discover and explain mathematical procedures and ideas. Practicing procedures is still important to the teachers, but mathematical reasoning and inquiry have increased in relative importance.

Fennema et al.'s 1996 longitudinal study of the CGI professional development program described teachers' changing beliefs and instruction as the result of their learning about the ways in which students learn addition and subtraction. The teachers in the CGI study experienced changes in belief “from demonstrating procedures to helping children build on their mathematical thinking by engaging them in a variety of problem-solving situations and encouraging them to talk about their mathematical thinking” (p. 403). This change in belief was also expressed by the teachers in this study in their shifts towards beliefs that focused on mathematical reasoning and on inquiry.

Teachers’ changing pedagogical beliefs were also a focus of Sowder et al.’s (1998) study of a professional development training. The teachers in that study were trained in mathematics content, pedagogy, and student learning. The pedagogical goals of teachers in the Sowder et al. study, similar to this study, became more focused on the conceptual understandings of students in addition to procedural skill development. Additionally, teachers in the Sowder et al. study changed the ways in which they asked students questions in order to get at student understandings and changed their views about instructional materials expressing a desire to find instructional materials that were more focused on the purposeful development of conceptual understanding. Sowder et al.’s findings about questioning and conceptual understanding were also present in this study, but only Alex expressed similar beliefs about instructional materials.

Wood et al. (1991) studied one teacher as she changed her classroom practices and beliefs about learning and teaching while participating in a project aimed at implementing researcher-created materials “based on constructivist views of learning” (1991, p. 587). The results of that study matched the results of this one, but, as with the CGI study and Sowder et al. (1998), the path to those results was different. In this study, teachers learned to teach by participating in learning. In Wood et al., the teacher learned to teach through participation in teaching using materials aligned to constructivist ideas and through participation in project meetings in which the materials, along with teacher questions and concerns, were discussed. In both studies, the teachers' beliefs changed in nearly identical ways. Wood et al.'s "analyses indicated that changes occurred in [the teacher's] beliefs about the nature of (a) mathematics from rules and procedures to meaningful activity, (b) learning from passivity to interacting and communicating, and (c) teaching from transmitting information to initiating and guiding students' development of knowledge" (1991, p. 587). These three changes correspond with the three changes in belief from this study to beliefs that were more focused on mathematical reasoning, more focused on inquiry, and more student-centered.

The teachers in this study experienced changes in beliefs that align with other research findings from studies that examine changes in teacher beliefs in response to professional development. In contrast to the three studies discussed above, the professional development in this study focused solely on teachers' mathematical content knowledge, not knowledge of students as learners or knowledge of pedagogy. These findings indicate that these changes in belief can result without instruction in student learning or in pedagogy.

Learning to Teach through Participation in Learning

The teachers in this study stated that they learned content, learned pedagogy, and experienced a deepening of their beliefs about lesson quality as a result of their participation in the professional development. The data presented in Chapter IV showed that teachers' changes in content knowledge, in pedagogical knowledge, and in beliefs about quality lessons are all tied to the fact that teachers experienced the learning of mathematics through their participation in an apprenticeship of observation in methods of inquiry.[67]

UEM did not teach knowledge pedagogy or student learning explicitly; it taught content. Before the professional development, Alex, and to a lesser extent Leona, were the only teachers to express a substantial level of conceptual ideas about mathematics. After the professional development, every teacher expressed gaining an understanding of the reasoning behind the mathematics and the multiple ways problems can be approached. These teachers learned conceptual content from the professional development. Many (all but Ireane) expressed an ability to solve problems (i.e. arrive at a solution using a procedure) before the professional development but stated that they learned the rationale behind those procedures and alternative ways of approaching the problems. Teachers gained conceptual understanding.

More interesting is that the teachers learned about pedagogy and learners from the experience of learning the content and observing the teaching of mathematics content. The teachers described a deepening of their beliefs to beliefs that are more mathematical and focused on mathematical reasoning, that are more student-centered and focused on guiding students in their learning of mathematics, and that use pedagogical strategies of inquiry to develop conceptual understanding. These beliefs speak to all three circles of knowledge in the framework for understanding teaching and learning, and yet only one of the circles (mathematics content) was explicit in the professional development. These experienced teachers were able to learn about the

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[67] See page 176.

other two circles (teaching and learners), specifically in the areas where those circles overlap subject matter, through their experience learning mathematics. This means that instead of teachers' beliefs being pulled toward the mathematics circle, the teachers used their experience and apprenticeship to integrate their knowledge of learners, teaching, and mathematics more towards their new vision of professional practice in the center of the framework. Teachers directly credited their experience in the professional development with changing their beliefs. In the last chapter I called this *learning to teach through participating in learning*.

One of the research questions for this study is: How does the experience of learning mathematics content through inquiry change teachers' beliefs about what constitutes a "quality mathematics lesson?" There are two answers to this question. The first answer describes the changes in belief, while the second describes the process of change and what made that change happen.

Teachers' beliefs about what constitutes a "quality mathematics lesson" became more focused on mathematical reasoning, more focused on inquiry, and more student-centered. Teachers' belief statements described a new vision of professional practice in which teachers: guide students as they engage in discovering and discussing mathematics and connections among mathematical content, building conceptual understanding, and practicing procedures; create and maintain a learning environment that allows students to engage safely and fully in the learning process; use multiple strategies while guiding students, including strategies of inquiry, that lead the student to discover and reason about mathematics in addition to developing strategies for computation.

The experience of learning mathematics content through inquiry changes teachers' beliefs by providing an opportunity for teachers to learn to teach through participation in learning. This process followed a constructivist model of learning. The constructivist philosophy of learning applies not only to students, but to adult learners including teachers (e.g. Maher & Alston, 1990). "Teachers' histories with mathematics learning powerfully shape their sense of themselves as mathematical knowers as well as their dispositions toward the subject" (Ball, 1996, p. 37). Indeed, the instruction in the professional development required the teachers to learn anew. Schifter (1996b) argues that "the new mathematics pedagogy … can be enacted only if teachers construct for themselves practices appropriate to its principles" (p.1). Learning mathematics with conceptual understanding provided teachers with mathematical learning experiences that they can use as models (to use the teachers' term) for ways to teach in the future. In addition to understanding mathematics content at a more conceptual level, the teachers in this study learned how to teach mathematics through inquiry. Learning mathematics content through inquiry not only changes experienced teachers' content knowledge, it changes their pedagogical beliefs as well.

Analysis of teachers' changes in content knowledge and their descriptions of what they called modeling, in conjunction with discussions about my data with other doctoral students and professors, led me to look at my study as an examination of teacher learning. I recalled hearing the statement that "teachers teach as they were taught, not as they were taught to teach." In searching for this idea in the research literature I was led first to Lortie (1975/2002). Lortie said that teachers do not enter teacher training as blank slates, they enter with years of observations of teaching in action. He called this the *apprenticeship of observation*. During this apprenticeship students observe acts of teaching and develop conceptions about teaching. Lortie notes that this apprenticeship does not come with the ability to see the rationale behind teachers' pedagogical decisions and hence students often develop misconceptions about the acts of teaching. "What students learn about teaching, then, it is intuitive and imitative rather than explicit and analytical," Lortie writes. "It is based on individual personalities rather than pedagogical principles" (Lortie, 1975/2002, p. 62). Although Lortie's work has been criticized by Mewborn and Tyminski (2006) for his conclusion that teaching is impacted more by the apprenticeship of observation than by teacher training programs, the critics agree with Lortie that "pre-service teachers have ideas and beliefs about what it means to teach mathematics when they enter a preparation program" (Mewborn & Tyminski, 2006, p. 32).

Unlike the students referenced in Lortie's (1975/2002) apprenticeships of observation, the teachers in this study were not just participating as *students*, they were participating as *experienced teachers*. They already had acquired pedagogical knowledge and expertise. Their observations were the observations of experts, and experts notice things differently than novices (Bransford, Brown, & Cocking, 2000). Although the teachers in this study were not all experts at mathematical pedagogy, they had achieved competence in general pedagogy. Therefore, they could apply their knowledge of general pedagogy to mathematics pedagogy when they saw the ways in which the instructor used her pedagogical content knowledge. The data in this study supports that experienced teachers can and do learn about pedagogy through their apprenticeships of observation, and that those apprenticeships are not merely of observation, but include an understanding of the instructor's actions. Experienced teachers *do* learn to teach through participating in learning.

Limitations

The participants in this study were all White Caucasian females. These women were experienced, elementary teachers teaching in grades three through eight. Each participant was asked to participate in the study after she signed up for the UEM professional development training. This is a volunteer, convenience sample and is a limitation of the study. These teachers wanted to learn more mathematics. It would be beneficial to repeat this study with teachers who had not chosen to learn more mathematics.

Data was collected over a six-month period of one school year. This study provides no information about whether or not teachers' experience will result in long-term changes in belief. Additional study over a longer timeframe would allow for the examination of long-term changes in belief.

This study did not examine teachers' classroom instruction. The findings of Cohen's (1990) study warn of the dangers of assuming that teacher report of changes to classroom instruction would match researcher observations. Although teachers submitted lesson plans and often shared examples from their classroom instruction, these sources of data were all collected via the teachers themselves and may have flaws typical of self-report data.

This study examined teacher beliefs about and teacher learning of mathematics and the teaching of mathematics. Fennema and Franke (1992) assert that "teacher knowledge cannot be separated from the subject matter being investigated, from how that subject matter can be represented for learners, from what we know about students' thinking in specific domains, or from teacher beliefs" (p. 161). The results of this study can only be generalized to teacher beliefs about and teacher learning of mathematics, not other subject areas.

The professional development training used in the study taught mathematics content. Because the instructor and the teachers participating were all practicing teachers, there were instances where pedagogical discussions arose. The instructor often curtailed those discussions. In a few instances, I stepped in and asked teachers to stop discussing pedagogy. The participants did have both short (~10 minute) bathroom breaks and longer lunch breaks. Some participants worked in the same school as each other.[68] It is possible, perhaps probable, that the teachers in the study did discuss pedagogy outside of the professional development, either with other teachers in the professional development training or other teachers with whom they have professional contact. This provides an additional limitation in that teacher changes in belief may well be influenced by outside discussions of pedagogy.

Many studies, including the widely-referenced Hawthorne studies, have called attention to the idea that when people participate in studies, the participation itself may cause changes in behavior (Cook, 1967). This is a known and accepted limitation of many qualitative methods. One way to increase construct validity and increase the likelihood that the observed changes are due to factors other than study participation is to use multiple sources of data.[69] This study examined teachers' changes in belief about quality lessons through teacher lesson plans and teacher statements in reflections,

[68] Bethan and Marie worked in the same school.
[69] See also Issues of Validation and Reliability section in Chapter III for an in-depth discussion of this issue.

journals, and interviews. The varied nature of these data sources increases the likelihood that the professed changes in beliefs are indeed belief changes.

In addition, Ernest's (1989) cycle of mathematics teachers' planning, teaching, and reflecting suggests that beliefs may both change and be changed by teachers' experiences with instruction. Teachers in this study shared changes in their classroom lessons as evidence of their changing beliefs. Ernest (1989) would argue that these changes in instruction may influence teachers' beliefs and those belief changes may, in turn, influence teachers' instruction. Therefore, changes in teacher belief become part of a feedback loop reinforcing and encouraging further change in belief. Note that Ernest's (1989) cycle also includes reflection. Teachers in this study reflected on their lessons, their experience learning mathematics, and their beliefs about quality mathematics lessons. It is quite possible that this reflection influenced the results of this study. Teachers learned to teach through participation in the learning of mathematics and reflecting on that process. What is not possible to determine is how much of that reflection was driven by participation in the study versus by teachers' natural reflection on their own learning.

Conclusion

This study answered two research questions: (1) What do experienced K-8 teachers believe constitutes a "quality mathematics lesson?" (2) How does the experience of learning mathematics content through inquiry change teachers' beliefs about what constitutes a "quality mathematics lesson?"

Teachers shared a vision of professional practice that described quality mathematics lessons, and teachers' roles in creating them. In summary, they reported the following:

> Teachers guide students as they engage in discovering and discussing mathematics and connections among mathematical content, building conceptual understanding, and practicing procedures. Teachers create and maintain a learning environment that allows students to engage safely and fully in the learning process. Teachers use multiple strategies, including strategies of inquiry, while guiding students. Those strategies may differ depending on the student, but all of the strategies help lead the student to discover and reason about mathematics in addition to developing strategies for computation.

This new vision incorporates many of teachers' initial views about lesson quality, but has become more focused on mathematical reasoning, more focused on inquiry, and more student-centered. Their vision has also become more integrated and centered on the intersection between mathematics, teaching, and learning.

The experience of learning mathematics content through the use of inquiry gave teachers the opportunity to learn to teach mathematics through their participation in the learning of mathematics. The teachers in this study were students of mathematics, but they were not just participating as *students*, they were participating as *experienced teachers*. This experience allowed them to view and understand the rationale behind the instructor's pedagogical moves. This illustrates that not only did the teachers in this study change their beliefs about quality instruction, but that experienced teachers can and do learn about teaching through participating in learning.

This study added to the literature in two areas: teacher beliefs about lesson quality, and the process by which teachers learn to teach through participation in learning. Together the findings in these two areas show the breadth of findings possible from a study on something as specific as teacher beliefs about quality mathematics lessons. Perhaps the largest lesson I learned in conducting this study and analyzing the results, is not to pigeonhole or limit my data. I studied teacher beliefs about quality mathematics lessons; and I learned so much more. Just as the teachers in this study learned to teach by participating in learning, I learned to allow myself to see the multiple things my data was telling me, rather than just what my data was telling me about what I thought I was studying.

Chapter VI: Implications

The teachers in this study changed their beliefs about quality mathematics lessons, and did so through participating in the learning of elementary mathematics content. The results of this study have implications for understanding research on teacher beliefs, for in-service teacher training in mathematics teaching, and for improving student achievement in mathematics.

This study found that teachers' beliefs about quality mathematics lessons changed; they became more focused on mathematical reasoning, more focused on inquiry, and more student-centered. Through their discussions about their beliefs, teachers revealed that their meanings of what it meant to be engaged or to have mathematical discussions were notably different than what their meanings had been when the teachers first expressed these ideas at the beginning of the study. Had this study only examined teachers' pre-training beliefs, these differences in meaning may not have been discovered. Any examination of teacher beliefs must examine not only, for example, if teachers believe that students should be engaged in learning, but also what those teachers mean by the term engagement. Conducting or interpreting research on teacher beliefs without an understanding of what teachers mean by their beliefs could vastly misrepresent the status of teachers' beliefs and of the congruence of those beliefs with definitions of quality from the literature.

This study has further implications for teacher training and professional development. For experienced teachers, the act of learning is a teaching experience. When experienced teachers learn something new, they bring their lens of teaching to their learning; they bring their pedagogical expertise to their apprenticeship of observation. This has important implications for in-service teacher training. If we want teachers to teach in certain ways, it is not enough to tell them how to teach. The teachers in this study believed that student engagement was important, but they didn't really understand what engagement meant until they experienced it themselves. Their past experiences with standards, teaching materials, and pedagogical trainings were not enough. These teachers had to experience being engaged *in mathematics* in order to understand what engagement in the mathematics classroom looked like. If we want teachers to truly change their teaching, we must teach them content using the methodologies that we want them to use with their students. If we do this, the teachers will learn to teach mathematics through their participation in the learning of mathematics.

Many mathematics professional development opportunities focus on the teaching of mathematics content and the methods by which one should teach that content. The results of this study imply that teaching content using the methods that one wishes teachers to use in their classrooms may also teach them those methods. Every time we teach content to experienced teachers, we are also teaching pedagogy. If we teach mathematics to teachers using traditional methods of instruction, we continue the status quo and can only expect that we will find the same result, that teachers teach using traditional methods. Instead, if we teach experienced teachers mathematics using methods of inquiry, teachers will also learn inquiry methods of teaching. This does not mean that experienced teachers will necessarily have the skill, support, or freedom to implement those inquiry methods in their classrooms; but they do learn them. The focus of further professional development can then shift to recognizing and removing barriers to the use of these methods in classroom instruction, and to increased skill in implementing them.

In addition to learning inquiry methods of teaching, the teachers in this study changed their beliefs about what quality mathematics teaching looks like. Their beliefs about quality mathematics lessons became more aligned with the research on lesson quality. This change in beliefs has implications for changing teachers’ teaching in classrooms. Although this study did not examine teachers’ classrooms for change, change in beliefs may be a necessary component of change in practice. As Cooney (1994) states: “ultimately, any attempt to reform the teaching of mathematics is an exercise in the adaption from what we are *able to do* to what we *want to do*” (p. 9). This combination of learning methods of inquiry and changing beliefs about the role of the teacher is a powerful indicator that the experience of learning content may well change teachers’ classrooms.

This study has implications not only for work with experienced teachers, but also for student learning. Student achievement is perhaps the most important goal of education. Understanding teacher beliefs about quality lessons, how to change teachers’ beliefs about lesson quality, and how those beliefs match the literature’s views on quality mathematics lessons allows us to determine where to focus our reform efforts. This study tells us that teachers’ beliefs about quality lessons do not routinely match the literature’s definition of quality, but that when teachers are taught mathematics using methods of inquiry, their beliefs can change to beliefs that better match the literature on quality mathematics lessons and on quality teaching for student achievement. The research on the ways in which beliefs limit the expression of knowledge in the classroom (e.g. Aguirre & Speer, 2000; Bray, 2011; Provost, 2013; Raymond, 1997) tells us that efforts to train teachers in teaching methods that improve student achievement without first changing teachers' beliefs about those methods may be missing the point.

If we can use the findings of this study, and other similar studies, to change the ways in which we provide teacher training to in-service teachers, we have the potential to change teaching as a profession now and into the future. Today's teachers are providing apprenticeships of observation for tomorrow's teachers. The faster we are able to improve mathematics teaching quality, the more future teachers will have learned mathematics through high quality instruction. This cannot be overstated. Not only can we improve the achievement of today's students of mathematics through providing opportunities for teachers to experience learning mathematics in a new way; but we can also improve student achievement for generations to follow by providing observational apprenticeships for tomorrow's teachers in quality instructional practices.

References

Aguirre, J., & Speer, N. M. (2000). Examining the relationship between beliefs and goals in teacher practice. *Journal of Mathematical Behavior*, *18*(3), 327-356.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal*, *90*(4), 449–466.

Ball, D. L. (1996). Connecting to mathematics as part of learning to teach. In D. Schifter (Ed.), *What's happening in math class, volume 2: reconstructing professional identities* (pp. 36-45). New York: Teachers College Press.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *29*(3), 14-22, 43-46.

Ball, D. L., Thames, M. H., & Phelps, G. C. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. doi:10.1177/0022487108324554

Banilower, E. R., Smith, P. S., Weiss, I. R., Malzahn, K. A., Campbell, K. M., & Weis, A. M. (2013). *Report of the 2012 National Survey of Science and Mathematics Education*. Chapel Hill, NC: Horizon Research, Inc.

Battista, M. T. (1990). Fifth graders' enumeration of cubes in 3D arrays: Conceptual progress in an inquiry-based classroom. *Journal for Research in Mathematics Education. 30*(4), 417-448.

Bereiter, C. (2006) Reflections on depth. In K. Leithwood, P. McAdie, N. Bascia, & A. Rodrigue (Eds.), *Teaching for deep understanding: What every educator should know* (pp. 11-16). Tousand Oaks, CA: Corwin Press.

Berliner, D. C. (1991). Educational psychology and pedagogical expertise: New findings and new opportunities for thinking about training. *Educational Psychologist*, *26*(2), 145–155.

Berliner, D. C. (2004). Describing the behavior and documenting the accomplishments of expert teachers. *Bulletin of Science, Technology and Society*, *24*(3), 200–212. doi:10.1177/0270467604265535

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal*, *17*(2), 39–68.

Beswick, K. (2007). Teachers' beliefs that matter in secondary mathematics classrooms. *Educational Studies in Mathematics*, *65*(1), 95–120. doi:10.1007/s10649-006-9035-3

Borko, H., & Livingston, C. (1989). Cognition and improvisation: Differences in mathematics instruction by expert and novice teachers. *American Educational Research Journal*, *26*(4), 473–498. doi:10.3102/00028312026004473

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school* (Expanded Ed.). Washington, DC: National Academies Press.

Bransford, J. D., Darling-Hammond, L, & LePage, P. (2005). Introduction. In L. Darling-Hammond & J. D. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should know and are able to do.* (pp. 1-39). San Francisco: Jossey-Bass.

Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education*, *42*(1), 2–38.

Brown, C. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws, (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). Reston, VA: National Council of Teachers of Mathematics.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis.* Thousand Oaks, CA: Sage Publications.

Chi, M. (2005). Commonsense conceptions of emergent processes: Why some misconceptions are robust. *Journal of the Learning Sciences*, *14*(2), 161-199. doi:10.1207/s15327809jls1402_1

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, *12*(3), 311–329.

Cook, D. L. (1967). *The impact of the Hawthorne Effect in experimental designs in educational research: Final report* (Report No. P-1757). Retrieved from ERIC database (Accession No. ED021308): http://www.eric.ed.gov/contentdelivery/servlet/ERICServlet?accno=ED021308

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16*(5), 324-336.

Cooney, T. J. (1994). Teacher education as an exercise in adaption. In D. B. Aichele (Ed.), *Professional development for teachers of mathematics: 1994 yearbook* (pp. 9-22). Reston, VA: National Council of Teachers of Mathematics.

Creswell, J. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Creswell, J. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Upper Saddle River, NJ: Pearson.

Darling-Hammond, L., & Bransford, J. (Eds.) (2005). *Preparing teachers for a changing world: What teachers should know and are able to do.* San Francisco: Jossey-Bass.

Davis, B., & Renert, M. (2014). *The math teachers know: Profound understanding of emergent mathematics.* New York: Routledge.

Davis, R. B., Maher, C. A., & Noddings, N. (Eds.) (1990). *Constructivist views on the teaching and learning of mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*(3), 181–199. doi:10.3102/0013189X08331140

Eisenhart, M., Borko, H., Underhill, R. G., Brown, C. A., Jones, D., & Agard, P. C. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education*, *24*(1), 8–40.

Ernest, P. (1988, August). *The impact of beliefs on the teaching of mathematics*. Paper presented at Sixth International Congress of Mathematical Education, Budapest. Retrieved from http://people.exeter.ac.uk/PErnest/impact.htm

Ernest, P. (1989). The knowledge, beliefs, and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, *15*(1), 13-33.

Even, R., & Tirosh, D. (1995). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject-matter. *Educational Studies in Mathematics*, *29*(1), 1-20.

Fang, Z. (1996). A review of the research on teacher beliefs and practices. *Educational Research, 38*(1), 47-65.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*(4), 403–434.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). Reston, VA: National Council of Teachers of Mathematics.

Fosnot, C. T. (Ed.) (1996). *Constructivism: Theory, perspectives, and practice*. New York: Teachers College Press.

Gage, N. L. (2009). *A conception of teaching.* New York: Springer.

Garner, M. (2007). An Alternative Theory: Deep Understanding of Mathematics. *Measurement*, *5*(2), 170–173.

Goldin, G. A. (1990). Epistemology, constructivism, and discovery learning in mathematics. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.) (1990), *Constructivist views on the teaching and learning of mathematics* (pp. 31-47). Reston, VA: National Council of Teachers of Mathematics.

Green, T. F. (1971). *The activities of teaching*. New York: McGraw-Hill.

Grossman, P. L. (1992). Why models matter: An alternate view on professional growth in teaching. *Review of Educational Research*, *62*(2), 171–179. doi:10.3102/00346543062002171

Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher*, *15*(5), 5-12. doi:10.3102/0013189X015005005

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Porsmouth, NH: Heinemann.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*(4), 430–511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, *105*(1), 11–30. doi:10.1086/428763

Hollingsworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal*, *26*(2), 160–189. doi:10.3102/00028312026002160

Illustrative Mathematics. (n.d.). *Content standards: Kindergarten through grade 8*. Retrieved from http://www.illustrativemathematics.org/standards/k8

Kagan, D. M. (1990). Ways of evaluating teacher cognition: Inferences concerning the Goldilocks Principle. *Review of Educational Research*, *60*(3), 419–469. doi:10.3102/00346543060003419

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Koehler, M., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 115-126). Reston, VA: National Council of Teachers of Mathematics.

Learning Mathematics for Teaching (2008). *Mathematical knowledge for teaching measures: Elementary number concepts and operations.* Ann Arbor, MI: Authors.

Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*(1), 91-102. doi:10.1007/s10857-006-9006-8

Leinhardt, G. (1989). Math lessons: A contrast of novice and expert competence. *Journal for Research in Mathematics Education*, *20*(1), 52–75.

Leinhardt, G. & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247-271.

Liljedahl, P. (2010). Noticing rapid and profound mathematics teacher change. *Journal of Mathematics Teacher Education*, *13*(5), 411-423. doi:10.1007/s10857-010-9151-y

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage Publications.

Lortie, D. C. (2002). *Schoolteacher: A sociological study* (2nd Ed.). Chicago: University of Chicago Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates.

Maher, C. A. & Alston, A. (1990). Teacher development in mathematics in a constructivist framework. In R. B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 31-47). Reston, VA: National Council of Teachers of Mathematics.

Marlowe, B. A. & Page, M. L. (2005). *Creating and sustaining the constructivist classroom* (2nd Ed.). Thousand Oaks, CA: Corwin Press.

Marton, F., & Säljö, R. (1976). On qualitative differences in learning: I--outcome and process. *British Journal of Educational Psychology*, *46*(1), 4–11.

Mewborn, D. S., & Cross, D. I. (2007). Mathematics teachers' beliefs about mathematics and links to students' learning. In W. G. Martin, M. E. Strutchens, & P. C. Elliott (Eds.), *The Learning of Mathematics: Sixty-ninth yearbook* (pp. 259-269). Reston, VA: National Council of Teachers of Mathematics.

Mewborn, D. S., & Tyminski, A. M. (2006). Lortie's apprenticeship of observation revisited. *For the Learning of Mathematics*, *26*(3), 30–33.

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review*, *13*(2), 125–145.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Center for Teacher Effectiveness (2012). *Measuring MQI*. Retrieved 4/7/2012 from http://isites.harvard.edu/icb/icb.do?keyword=mqi_training&pageid=icb.page394700

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington DC: Author.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U. S. Department of Education. Retrieved from http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Nesher, P. (1986, April). *Are mathematical understandings and algorithmic performance related?* Invitational Address for the 1986 Annual Meeting of the American Educational Research Association, Division C-4. San Francisco, CA.

No Child Left Behind Act of 2001, Pub. L. No. 107-110, § 101 *et seq*., 115 Stat. 1439, and § 1119 *et seq*., 115 Stat. 1505 (2002). Available from http://www2.ed.gov/policy/elsec/leg/esea02/index.html

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, *62*(3), 307-332. doi:10.3102/00346543062003307

Patton, M.Q. (2002). *Qualitative research and evaluation methods*. (3rd ed.) Thousand Oaks, CA: Sage Publications.

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teacher's pedagogical content beliefs in mathematics. *Cognition & Instruction*, *6*(1), 1-40.

Phillips, D. C., & Soltis, J. F. (2009). *Perspectives on learning*. New York: Teachers College Press.

Provost, L. E. (2013). *The multifaceted nature of mathematics knowledge for teaching: Understanding the use of teachers' specialized content knowledge and the role of teachers' beliefs from a practice-based perspective* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3575981)

Putnam, R. T., Heaton, R. M., Prawat, R. S., & Remillard, J. (1992). Teaching mathematics for understanding: Discussing case studies of four fifth-grade teachers. *Elementary School Journal*, *93*(2), 213-228.

Putman, S. M. (2009). Grappling with classroom management: The orientations of preservice teachers and impact of student teaching. *The Teacher Educator*, *44*(4), 232–247. doi:10.1080/08878730903180226

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, *28*(5), 550-576.

Ross, J. A., McDougall, D., & Hogaboam-Gray, A. (2002). Research on reform in mathematics education, 1993-2000. *The Alberta Journal of Educational Research*, *48*(2), 122–138.

Sanders, W. L., & Horn, S. P. (1998). Research findings from the Tennessee Value-Added Assessment System (TVAAS) database: Implications for educational evaluation and research. *Journal of Personnel Evaluation in Education*, *12*(3), 247-256.

Savion, L. (2009). Clinging to discredited beliefs: The larger cognitive story. *Journal of the Scholarship of Teaching and Learning*, *9*(1), 81-92.

Schwartz, D. L., Bransford, J. D. & Sears, D. (2005). Efficiency and innovation in transfer. In J. Mestre (Ed.), *Transfer of learning from a modern multidisciplinary perspective* (pp. 1-51). Greenwich, CT: Information Age Publishing.

Shifter, D. (1996a). A constructivist perspective on teaching and learning mathematics. In Fosnot, C. T.. (Ed.) (1996), *Constructivism: Theory, perspectives, and practice* (pp. 73-91). Reston, VA: National Council of Teachers of Mathematics.

Schifter, D. (Ed.) (1996b). *What's happening in math class, volume 2: Reconstructing professional identities*. New York: Teachers College Press.

Schrum, L., Burbank, M. D., & Capps, R. (2007). Preparing future teachers for diverse schools in an online learning community: Perceptions and practice. *The Internet and Higher Education*, *10*(3), 204–211. doi:10.1016/j.iheduc.2007.06.002

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher , 15* (2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-22.

Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School, 12*(2), 88-95 (Reprinted from *Mathematics Teacher, 77*, 1976, pp. 20-26).

Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education*, *4*, 3-28.

Slavin, R. E., Lake, C., & Groff, C. (2009). Effective programs in middle and high school mathematics: A best-evidence synthesis. *Review of Educational Research*, *79*(2), 839–911. doi:10.3102/0034654308330968

Sowder, J. T., Phillip, R. A., Armstrong, B. E., & Schappelle, B. P. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction: A research monograph.* Albany, NY: State University of New York.

Stiler, G. M., & Philleo, T. (2003). Blogging and blogspots: An alternative format for encouraging reflective practice among preservice teachers. *Education*, *123*(4), 789–797.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*, 213–226.

Study of Instructional Improvement (n.d.). *Instrument dissemination workshop binder*. Received during training, June, 2010. Ann Arbor, MI: Learning Mathematics for Teaching Project.

Sugrue, C. (1997). Student teachers' lay theories and teacher identities: Their implications for professional development. *European Journal of Teacher Education*, *20*(3), 213–225.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). Reston, VA: National Council of Teachers of Mathematics.

Tracy, S. J. (2013). *Qualitative research methods: Collecting evidence, crafting analysis, communicating impact.* Malden, MA: Wiley-Blackwell.

Van Zoest, L. R., Jones, G. A., & Thornton, C. A. (1994). Beliefs about mathematics teaching held by pre-service teachers involved in a first grade mentorship program. *Mathematics Education Research Journal*, *6*(1), 37–55.

von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 19-29). Reston, VA: National Council of Teachers of Mathematics.

Wenglinsky, H. (2002, February). How schools matter: The link between classroom practices and student academic performance. *Education Policy Analysis Archives, 10*(2). Retrieved from http://epaa.asu.edu/epaa/v10n12

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge : An examination of research on contemporary professional development. *Review of Research in Education*, *24*, 173–209.

Wilson, P. S., Cooney, T. J., & Stinson, D. W. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers' perspectives. *Journal of Mathematics Teacher Education*, *8*(2), 83-111. doi:10.1007/s10857-005-4796-7

Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Education Research Journal, 28*(3), 587-616.

Yin, R. K. (1994). *Case study research: Design and methods* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Bibliography

Ambrose, R., Clement, L., Philipp, R., & Chauvot, J. (2004). Assessing prospective elementary school teachers ' beliefs about mathematics and mathematics learning: Rationale and development of a constructed-response-format beliefs survey. *School Science & Mathematics*, *104*(2), 56–69.

Beauchamp, C., & Thomas, L. (2009). Understanding teacher identity: An overview of issues in the literature and implications for teacher education. *Cambridge Journal of Education*, *39*(2), 175–189. doi:10.1080/03057640902902252

Bray, W. S. (2007). *A study of teacher transitions to a reform-based mathematics curriculum in an urban school: The interaction of beliefs, knowledge, and classroom practices* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UNI No. 3289020)

Calderhead, J. (1996). Teachers: Beliefs and knowledge. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 709–725). New York: Simon & Schuster.

Chval, K., Abell, S., Pareja, E., Musikul, K., & Ritzka, G. (2008). Science and mathematics teachers' experiences, needs, and expectations regarding professional development. *Eurasia Journal of Mathematics, Science & Technology Education*, *4*(1), 31–43.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics.* (pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.

Cooney, T. J. (1999). Conceptualizing teachers' ways of knowing. *Educational Studies in Mathematics*, *38*, 163–187.

Cooney, T. J., Shealy, B. E., & Arnold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics*, *29*(3), 306–333.

Eisner, E. W. (1991). *The enlightened eye: Qualitative inquiry and the enhancement of educational practice.* New York: Macmillan.

Ernest, P. (1991). Mathematics teacher education and quality. *Assessment & Evaluation in Higher Education*, *16*(1), 56-65.

Ernest, P. (1992). The nature of mathematics: Towards a social constructivist account. *Science and Education*, *1*, 89–100.

Feiman-Nemser, S. (2008). Teacher learning: How do teachers learn to teach? In M. Cochran-Smith, S. Feimen-Nemser, & D. J. McIntyre (Eds.), *Handbook of research on teacher education: Enduring questions in changing contexts* (3rd ed.). (pp. 697-705). New York: Routledge.

Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal*, *30*(3), 555–583. doi:10.3102/00028312030003555

Fishman, B. J., Marx, R. W., Best, S., & Tal, R. T. (2003). Linking teacher and student learning to improve professional development in systemic reform. *Teaching and Teacher Education*, *19*(6), 643–658. doi:10.1016/S0742-051X(03)00059-3

Garet, M. S., Porter, A. C., Desimone, L. M., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal*, *38*(4), 915–945. doi:10.3102/00028312038004915

Glaser, B. G., & Strauss, A.L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* New York: Aldine.

Hoyles, C. (1992). Mathematics teaching and mathematics teachers: A meta-case study. *For the Learning of Mathematics*, *12*(3), 32–44.

Jarvis, P., Holford, J., Griffin, C. (1998). *The theory and practice of learning.* Sterling, VA: Stylus Publishing.

Jaworski, B. (1998). The centrality of the researcher: Rigor in a constructivist inquiry into mathematics teaching. In A. R. Teppo (Ed.) (1998), *Qualitative research methods in education* (pp. 112-127). Reston, VA: National Council of Teachers of Mathematics.

Kang, H. J. (2012). *Identity development of preservice elementary teachers of mathematics from teacher education program to student teaching* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3522168)

Kennedy, M. M., Ahn, S., & Choi, J. (2008). The value added by teacher education. In M. Cochran-Smith, S. Feiman-Nemser, D. J. McIntyre, & K. E. Demers (Eds.), *Handbook of research on teacher education* (3rd ed., pp. 1247–1271). New York: Routledge.

Lerman, S. (1983). Problem-solving or knowledge-centered: The influence of philosophy on mathematics teaching. *International Journal of Mathematical Education in Science and Technology, 14*(1), 59-66.

Liljedahl, P., Rolka, K., & Rösken, B. (2007). Affecting affect: The reeducation of preservice teachers' beliefs about mathematics and mathematics learning and teaching. In W. G. Martin, M. E. Strutchens, & P. C. Elliott (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 319-330). Reston, VA: National Council of Teachers of Mathematics.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass.

Mira, R. V. (2008). *Teachers' beliefs regarding effective teaching strategies for American Indian students in mathematics* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3311469)

Nespor, J. K. (1985). *The role of beliefs in the practice of teaching: Final report of the teacher beliefs study* (R&D Report 8024). Austin, TX: R&D Center for Teacher Education, University of Texas at Austin.

Nisbet, S., & Warren, E. (2000). Primary school teachers' beliefs relating to mathematics, teaching and assessing mathematics and factors that influence these beliefs. *Mathematics Teacher Education and Development*, *2*, 34-47.

Peressini, D., Borko, H., Romagnano, L., Knuth, E., & Willis, C. (2004). A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics*, *56*(1), 67-96. doi:10.1023/B:EDUC.0000028398.80108.87

Philipp, R. A., Ambrose, R. C., Lamb, L. L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., Thanheiser, E., Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education*, *38*(5), 438–476.

Pirie, S. (1998). Toward a definition for research. In A. R. Teppo (Ed.), *Qualitative research methods in mathematics education (JRME Monograph No. 9)* (pp. 17-21). Reston, VA: National Council of Teachers of Mathematics.

Reason, P. & Rowan, J. (Eds.). (1981). *Human inquiry: A sourcebook of new paradigm research.* Chichester: John Wiley & Sons.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. P. Sikula, T. J. Buttery, & E. Guyton (Eds.), *Handbook of research on teacher education* (2nd ed., pp. 102–119). New York: Macmillan.

Sabers, D. S., Cushing, K. S., & Berliner, D. C. (1991). Differences among teachers in a task characterized by simultaneity, multidimensionality, and immediacy. *American Education Research Journal, 28*(1), 63-88.

Schatzman, L. & Strauss, A. (1973). *Field research: Strategies for a natural sociology.* Upper Saddle River, NJ: Prentice-Hall.

Shulman, L. S., & Shulman, J. H. (2004). How and what teachers learn: A shifting perspective. *Journal of Curriculum Studies*, *36*(2), 257–271. doi:10.1080/0022027032000148298

Silver, E. A., Mesa, V. M., Morris, K. A., Star, J. R., & Benken, B. M. (2009). Teaching mathematics for understanding: An analysis of lessons submitted by teachers seeking NBPTS certification. *American Educational Research Journal*, *46*(2), 501–531. doi:10.3102/0002831208326559

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114–145.

Simon, M. A. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking and Learning*, *8*(4), 359–371. doi:10.1207/s15327833mtl0804_1

Smylie, M. A. (1989). Teachers' views of the effectiveness of sources of learning to teach. *Elementary School Journal, 89*(5), 543-558.

Steffe, L. P., & Nesher, P. (Eds.) (1996). *Theories of mathematical learning*. Mahwah, NJ: Lawrence Erlbaum Associates.

Strauss, A. & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques.* Newbury Park, CA: Sage Publications.

Swafford, J. O., Jones, G. A., & Thornton, C. A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education , 28* (4), 467-484.

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education*, *30*(1), 89–110.

Van Zoest, L. R., & Bohl, J. V. (2005). Mathematics teacher identity: A framework for understanding secondary school mathematics teachers' learning through practice. *Teacher Development*, *9*(3), 315–345.

Weissglass, J. (1994). Changing mathematics teaching means changing ourselves: Implications for professional development. In D. B. Aichele (Ed.), *Professional development for teachers of mathematics: 1994 yearbook* (pp. 67–78). Reston, VA: National Council of Teachers of Mathematics.

White, P., Mitchelmore, M., Branca, N., & Maxon, M. (2004). Professional development: Mathematical content versus pedagogy. *Mathematics Teacher Education and Development*, *6*, 41–51.

Wood, T., Cobb, P., & Yackel, E. (1990). The contextual nature of teaching: Mathematics and reading instruction in one second-grade classroom. *Elementary School Journal, 90*(5), 497-513.

Appendices

Appendix A. Framework for Understanding Teaching and Learning

The framework used in the analysis of this study's data is the Framework for Understanding Teaching and Learning (Darling-Hammond & Bransford, 2005). Figure A - 1 maps aspects of the literature and the data to this framework.

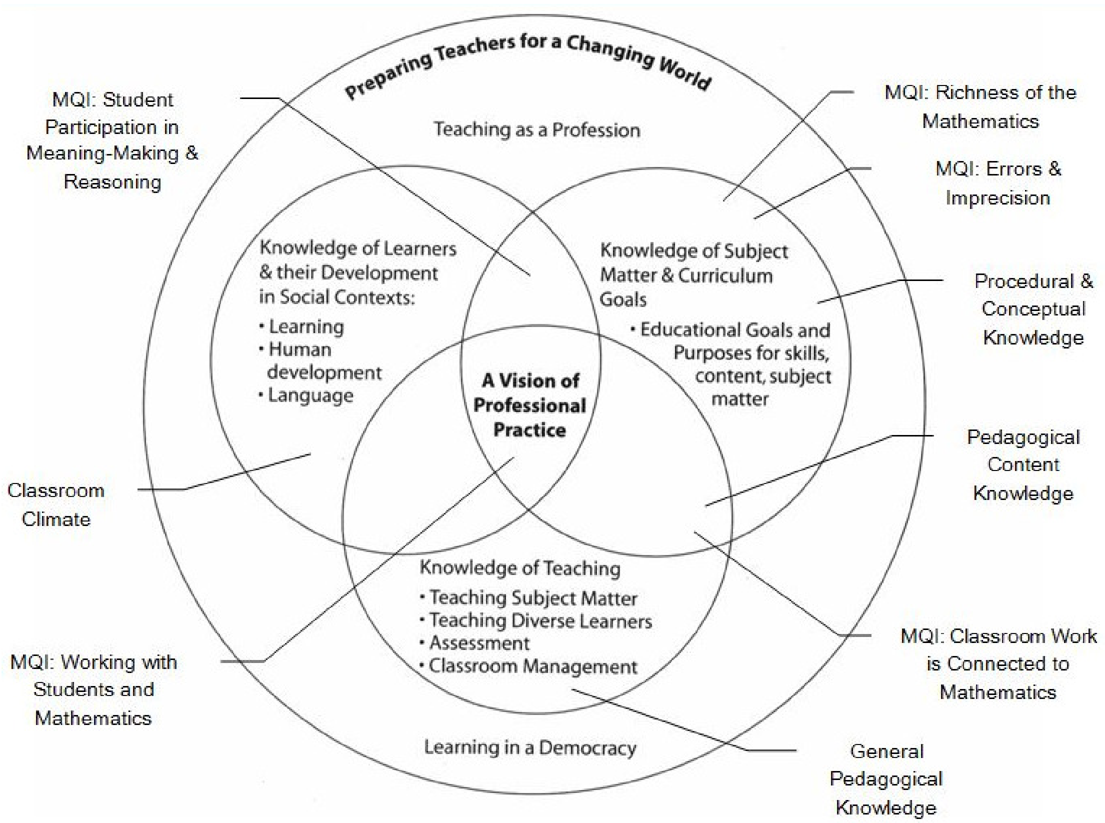

Figure A - 1. Aspects of the literature mapped to the Framework for Understanding Teaching and Learning. MQI refers to Mathematical Quality of Instruction (National Center for Teacher Effectiveness, 2012). Adapted from *Preparing Teachers for a Changing World* (p. 11), Edited by L. Darling-Hammond and J. Bransford, 2005, San Francisco: Jossey-Bass. Copyright 2005 by John Wiley & Sons, Inc. Adapted with permission.

Appendix B. Brief Biographical Descriptions of Participants

The information in this table is meant to be a quick reference for use when reading the dissertation.

Participant	Biographical Description
Alex MacMillan	Alex MacMillan is in her third full year of teaching[70] and has taught mathematics each year. She is currently teaching seventh and eighth grade students mathematics in a general education setting. She also collaborates with another teacher on the science instruction for these same students.
Bethan O'Connell	Bethan O'Connell teaches grades 6, 7, and 8 mathematics and social studies to students who have been identified as "at risk." She has been a special education teacher for 28 years and has taught math for exactly half of her career.
Ellyn Dustin	Ellyn Dustin currently teaches mathematics and reading to students in a pull-out setting removed from the general education setting. Two of her classes are fully individualized, and two parallel the general education curriculum. Ellyn has been teaching for eleven years and has taught math for four of those years. Before becoming a teacher, Ellyn worked in social work.
Ireane Croteau	Ireane Croteau is a Special Education teacher for grades 5-8. She provides supplemental mathematics, language arts, and science instruction for students. These students are in a pull-out setting for mathematics which is taught by another mathematics teacher. Ireane works with the students during their academic support class daily on the mathematics skills identified by the mathematics teacher. Ireane has been providing special education services at the middle school level for the last five years and previously spent ten years teaching life skills to high school students with more severe disabilities.

[70] Alex began teaching part way through a school year. Therefore, she had completed three years of teaching at the time of the study: one-half year, followed by two full years, and one-half of the school year during which the study took place.

Participant	Biographical Description
Leona Sawyer	Leona Sawyer teaches Title 1 Mathematics to third, fourth, and fifth grade students, which means that she provides small group instruction to students who have not been successful learning purely in the general education setting. Each student sees Leona 2-3 times per week for 25 minutes and she works with some students in additional intervention times through Response to Intervention (RTI). Students receive this instruction in addition to their regular classroom mathematics instruction. Leona is in her fourth year of teaching and has been providing mathematics Title 1 support for each of those four years.
Marie Gilmore	Marie Gilmore teaches four 6th grade math classes that are heterogeneously grouped and a state testing preparation class for approximately half of these same students. She has been teaching for 9 years, 7 of which have included mathematics. Before entering teaching, Marie worked in banking.
Mary Winship	Mary Winship is a Special Education teacher with 30 years of experience, 25 of which include mathematics teaching. She is currently teaching fifth and sixth grade students mathematics, reading, and writing in a pull-out setting and providing classroom support in the content areas. Her mathematics instruction is individualized for each student.
Sue Daniels	Sue Daniels is a veteran teacher with 30 years of teaching experience, 25 of those years teaching mathematics. She has experience teaching in elementary classrooms and as a preschool coordinator in addition to her current position teaching 6th through 9th grade mathematics in a virtual environment. She has been teaching these virtual mathematics courses for the last 5 years.

Appendix C. Data Collection and Analysis Flow Chart

CHANGE IN EXPERIENCED TEACHERS' PEDAGOGICAL BELIEFS[65] THROUGH LEARNING ELEMENTARY MATHEMATICS CONTENT

Research questions: (1) What do experienced K-8 teachers believe constitutes a "quality mathematics lesson?" (2) How does the experience of learning mathematics content through inquiry[66] change teachers' beliefs about what constitutes a "quality mathematics lesson?"

Pre-Training Phase

Demographic Info & Consent Forms
- verify eligibility
- teacher consent

Teachers submissions
- 1 lesson plan of "quality" lesson
- written reflection on lesson and quality lessons in general

Analysis (within-case)
- emergent themes from lessons and reflections

Interviews
- lessons
- beliefs about quality
- clarify/expand on written reflections

Analysis (within- and cross-case)
- emergent themes from submissions and interviews
- compare to existing literature

Participant Verification
- within-case

Training Phase

Professional Development
- pre-assessment of MKT
- post-assessment of MKT (not analyzed until end of study)

Teacher submissions
- 1 lesson plan of "quality" lesson with written reflection
- at least 3 journal entries regarding their thoughts on quality lessons as they participate in professional development

Analysis (within- and cross-case)
- emergent themes from teacher submissions
- pre-assessment data in light of pre-training and training phase data
- compare to existing literature

Post-Training Phase

Teachers submissions
- 1 lesson plan of "quality" lesson
- written reflection on lesson, changes in lesson choices and quality lessons in general

Analysis (within- and cross-case)
- emergent themes adding post-training data
- compare to existing literature

Interviews
- lesson plans from all three phases
- clarify/expand on and written reflections
- if/how beliefs have changed

Analysis (within-case)
- emergent themes adding data from interviews
- compare to existing literature

Participant Verification
- within-case

Analysis (cross-case)
- emergent themes with data from member checks
- MKT post-assessments
- compare to existing literature

Participant Verification
- cross-case
- online focus group

Analysis (cross-case)
- check and revise analysis based on final participant verification

[65] Beliefs are "anything the individual regards as true" (Beswick, 2007).
[66] Inquiry, in this study, describes the investigation of mathematical ideas (Schifter, 1996a).

Appendix D. UEM Advertising Flyer

Teachers were solicited for participation in the professional development used in the training phase of the study. This flyer was distributed in order to recruit participants.

Appendix E. Participant Consent Form

Consent Form for Participation
in a Research Study

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193

Study Title: Teacher Beliefs about Quality Mathematics Lessons

Introduction

You are being asked to participate in a research study conducted by Ann Gaffney, a doctoral student at Rivier University. This consent form will give you information you will need to help you decide whether or not you would like to participate. It will also describe what you will be asked to do and any known risks or inconveniences of participation. You should feel free to ask questions. If you have more questions later, Ann Gaffney, the person responsible for this study, (agaffney@rivier.edu or 603-490-7193) will discuss them with you. If you decide to participate, you will be asked to sign this form and a copy will be provided to you.

Why is this study being done and why am I invited to participate?

Ann Gaffney is interested in how experienced K through 8 mathematics teachers define quality mathematics lessons and the ways your beliefs about what makes a quality lesson may change (or not) as you learn more mathematics. This knowledge will help us understand the complicated relationship between what math teachers know, believe, and do. You are being asked to participate because of your experience teaching mathematics and because you have signed up for the Understanding Elementary Mathematics professional development program.

What are the study procedures? What will I be asked to do?

If you decide to take part in this study, here is what you will be asked to do:

1. Submit three mathematics lesson plans that you have used or intend to use in your classroom and a written reflection about each lesson plan.
2. Keep a journal about your thoughts and ideas about quality mathematics lessons as you participate in the study.
3. Participate in two individual interviews, each lasting approximately one hour, to take place at a location convenient to you. These interviews will be audiotaped.
4. Participate fully in the Understanding Elementary Mathematics professional development program. In addition, your scores from the pre- and post-tests used in

the Understanding Elementary Mathematics professional development program will be used in this study.

5. Verify the conclusions from this study. That means you will asked to read descriptions of your beliefs and correct any misunderstandings or misinterpretations of what you have said. These conversations will take place either in person, via live videoconference (such as Skype), or in an online discussion forum.

What are the risks or inconveniences of participating in this study?

This study collects information and statements from you that may reveal your identity. Precautions are taken to protect your identity (see below). There are no other risks to you because of your participation in the research study. There may be inconveniences, such as the time commitment involved. You can expect to spend a total of 12-15 hours on tasks associated with this study (not counting the time spent participating in the Understanding Elementary Mathematics professional development program).

What are the benefits of participating in this study?

Upon completion of the study, you will receive a certificate of participation in professional development. This certificate will not state that you participated in a research study, but will state that you participated in 15 hours of professional development called “Reflecting on Mathematics Practice and Quality Mathematics Lessons.” This professional development certificate for your participation in the study will only be awarded if you complete all parts of the study. Your participation in the Understanding Elementary Mathematics professional development program is separate from your participation in this study and withdrawing from the study will in no way impact your participation in the Understanding Elementary Mathematics professional development program. Upon request, you will be given a summary of the results of the study.

Will I receive payment for participation? Are there costs to me?

You will not be paid for your participation. There are no costs to you for participating in this study.

How will my identity and information be protected?

All data and records associated with your participation in this research will be kept confidential. If you choose to participate, you will choose a pseudonym that you will use on all submitted information and in audiotaped conversations. Your name and all other personally identifying information will be kept confidential; only Ann Gaffney will have access to that information. The sheet that connects your real name to your pseudonym will be stored in a locked cabinet separate from all other study data; at the completion of this study, those sheets will be destroyed. All other records generated from this study will be kept in a locked file cabinet and may be used in future research.

By signing this form, you are voluntarily agreeing to participate in this research study and are giving permission to utilize this data in research report(s) wherein your identity will be kept confidential.

Can I stop being in the study and what are my rights?

The decision to participate in this study is up to you. You do not have to participate in this study. If you decide to participate but then change your mind, you may quit at any time. Whatever you decide will in no way impact your participation in the Understanding Elementary Mathematics professional development program. If you wish to quit, simply inform Ann Gaffney (agaffney@rivier.edu or 603-490-7193) of your decision. At that time, any information collected from you will be removed from the study and destroyed.

Whom do I contact if I have questions about the study?

If you have any questions about this study or if you have a research-related problem, you may contact Ann Gaffney at the number/email listed at the top of this form. If you have any questions concerning your rights as a research participant, you may contact Dr. Jerome Rekart, chair of the Rivier University Research Review Board (RRB) at 603-897-8270.

Documentation of Consent:

I have read this consent form. The goals of the study and possible risks and benefits have been explained. All of my questions have been answered. My signature on this form means that I understand the information and that I voluntarily agree to participate in the study described above.

________________________	____________________	__________
Participant Signature:	Print Name:	Date:
________________________	____________________	__________
Ann Gaffney, Rivier University	Print Name:	Date:

Appendix F. Lesson Plan and Written Reflection Directions

First Lesson Plan and Reflection

Teacher Submissions
First Lesson Plan and Reflection

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193
Study Title: Teacher Beliefs about Quality Mathematics Lessons

Dear ___________________,

Thank you for participating in this study regarding teacher beliefs about quality mathematics lessons. Notice that I have used your pseudonym in this communication. All submissions are part of your participation in this research study and your confidentiality will be protected. This is the first of three lesson plans and reflections that you will be asked to submit as part of your participation in this study.

Step One: Choose a quality mathematics lesson that you have used with your students. It does not need to be a lesson you write, but it should be one that you consider high quality.

Step Two: Submit a lesson plan for this lesson (or use a write-up that you already have). If this lesson uses a textbook, please photocopy the applicable pages of the text. If this lesson uses worksheets, visuals, etc., please include copies of those as well.

Step Three: Complete the written reflection questions below.

Step Four: Send all these materials to me either electronically (agaffney@rivier.edu) or by mailing a hard copy (Ann Gaffney, 28 Tokanel Dr., Londonderry, NH 03053). **Please submit your lesson plan and reflection before your scheduled interview time BEFORE beginning Understanding Elementary Mathematics.** Thank you!

Lesson Background:

1. With what grade level have you used this lesson? Describe the type(s) of students with whom you used this lesson (e.g. heterogeneous regular education classroom, advanced course, etc.).

Reflection Questions:

2. Why did you select this lesson to submit?

3. What characteristics does this selection have that make it an example of a high quality lesson?

4. Is there anything about this lesson that you would change? How would these changes improve the quality of the lesson?

Second Lesson Plan and Reflection

Rivier UNIVERSITY

Teacher Submissions
Second Lesson Plan and Reflection

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193

Study Title: Teacher Beliefs about Quality Mathematics Lessons

Dear _____________________,

Thank you for your continued participation in this study regarding teacher beliefs about quality mathematics lessons. All submissions are part of your participation in this research study and your confidentiality will be protected. This is the second of three lesson plans and reflections that you will be asked to submit as part of your participation in this study.

Step One: Choose a quality mathematics lesson that you *intend to use* with your students. It does not need to be a lesson you write, but it should be one that you consider high quality.

Step Two: Submit a lesson plan for this lesson. If this lesson uses a textbook, please photocopy the applicable pages of the text. If this lesson uses worksheets, visuals, etc., please include copies of those as well.

Step Three: Complete the written reflection questions below.

Step Four: Send all these materials to me either electronically (agaffney@rivier.edu), handing them to me in class, or by mailing a hard copy (Ann Gaffney, 28 Tokanel Dr., Londonderry, NH 03053). **Please submit your lesson plan and reflection between March 13th and March 20th.** Thank you!

Lesson Background:

1. With what grade level do you intend to use this lesson? Describe the type(s) of students for whom you selected this lesson (e.g. heterogeneous regular education classroom, advanced course, etc.).

Reflection Questions:

2. Why did you select this lesson to submit?

3. What characteristics does this selection have that make it an example of a high quality lesson?

4. What would you change about this lesson to improve the quality of the lesson?

5. Think back to the first lesson you submitted. How does the quality of this lesson compare to the quality of the first lesson you submitted? What accounts for the differences in quality?

Third Lesson Plan and Reflection

Rivier UNIVERSITY

Teacher Submissions
Third Lesson Plan and Reflection

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193

Study Title: Teacher Beliefs about Quality Mathematics Lessons

Dear ______________________,

Thank you for your continued participation in this study regarding teacher beliefs about quality mathematics lessons. All submissions are part of your participation in this research study and your confidentiality will be protected. This is the third and final lesson plan and reflection that you will be asked to submit as part of your participation in this study.

Step One: Choose a quality mathematics lesson that you *intend to use* with your students. It does not need to be a lesson you write, but it should be one that you consider high quality.

Step Two: Submit a lesson plan for this lesson. If this lesson uses a textbook, please photocopy the applicable pages of the text. If this lesson uses worksheets, visuals, etc., please include copies of those as well.

Step Three: Complete the written reflection questions below.

Step Four: Send all these materials to me either electronically (agaffney@rivier.edu) or by mailing a hard copy (Ann Gaffney, 28 Tokanel Dr., Londonderry, NH 03053). **Please submit your lesson plan and reflection between March 24th and April 7th.** Thank you!

Lesson Background:

1. With what grade level do you intend to use this lesson? Describe the type(s) of students for whom you selected this lesson (e.g. heterogeneous regular education classroom, advanced course, etc.).

Reflection Questions:

2. Why did you select this lesson to submit?

3. What characteristics does this selection have that make it an example of a high quality lesson?

4. What things did you consider when creating/selecting this lesson? Why did you decide to submit this particular lesson over others?

5. Have the lessons you submitted changed over time? Please describe the nature of the change, if any.

Appendix G. Interview Protocol

The questions listed in this section will be asked of all participants. Additional questions may be added for purposes of clarification or elaboration. If an interviewee has already answered a particular question during the course of the interview, that question may be skipped.

Each interview question below is followed by the letter B, D, I, or C. Questions followed by the letter B are background questions. D questions intended to elicit teachers' definitions of "quality mathematics lessons." Questions aimed at eliciting responses about instructional goals and strategies are marked with the letter I. Questioned marked with the letter C are questions that ask participants to examine changes in some aspect of their beliefs.

Pre-Training Interview

1. When you were a student, were there any math teachers that you thought were particularly good? [B]
    a. What characteristics made these teachers exceptional? [D]
    b. What strategies did these teachers use to teach mathematics? [I]
2. How would you describe yourself as a mathematics teacher? [B]
3. What sorts of things are important to you as a math teacher? [B, D]
4. What do you think of when you hear the phrase "quality mathematics lesson"? [D]
5. What characteristics do you think good math lessons should have? [D]
6. What methods do you believe help students to learn mathematics best? [D, I]

7. Where do you find your mathematics lessons; do you create lessons on your own, collaboratively with peers, or find them from other sources? [B]
    a. What sources? [If applicable. B]
    b. Why those sources? [If applicable. B]
8. What other resources do you use when planning mathematics lessons? [B]
9. What goals do you have for your mathematics students? [I]
10. What strategies do you use to help your students learn mathematics? [I]
11. What teaching methods do you find most effective with your students? [I]
12. What are your beliefs regarding mathematics lesson quality? [D]
13. When you submitted your lesson plan, you wrote about ______ as one thing that makes your lesson high quality. What other qualities make lessons particularly good? [D]
14. When you submitted your lesson plan, you said that you would change _______ about your lesson. Are there reasons you didn't do that? [I]
    a. Are there obstacles that stood in the way of you changing the lesson? [I]

Post-Training Interview

1. What characteristics do great math lessons have in common? [D, I]
2. What do quality mathematics lessons ask students to do? [D, I]
3. How would you define the phrase "quality mathematics lesson?" [D]
4. Has your definition of quality changed? In what ways? [D, C]
5. What are your beliefs regarding mathematics lesson quality? [D]
6. In your journal entries you described the ways in which Understanding Elementary Mathematics changed/reinforced your beliefs about high quality

mathematics lessons. To summarize, you said: __________. Could you tell me more about this? [D, C]

7. What are your favorite teaching methods? [I]
8. What teaching methods help students learn mathematics best? [I]
9. Have your teaching methods changed since your participation in Understanding Elementary Mathematics? How? [I, C]
10. I am interested in your thoughts on the structure of mathematics lessons.
    a. What structure would an excellent mathematics lesson follow? [I]
    b. What key components would it have? [I]
11. I have with me the first lesson plan you submitted. I am interested in your thoughts on this lesson now. [Give time for the participant to review the lesson.]
    a. What do you think about this lesson? [D]
    b. Are there things you would change about this lesson now? [D, C]
    c. How would you change those things? [If applicable. I, C]
    d. What would these changes accomplish? [If applicable. D, I]
    e. Is there an event or situation that made you change your views on this lesson? [If applicable. C]
12. [Repeat the above question with the second and third lesson plans submitted, but stop if the process becomes repetitive and no new information is being gathered.]

13. When you submitted your third lesson plan, you indicated that your lessons have changed over time in these ways: ________. Why do you think this happened? [I, C]
14. [This question was added to the protocol later in the study.] You took a pre- and post-test during Understanding Elementary Mathematics. How did you feel about taking those tests? Do you feel that your scores were an accurate representation of your abilities?

Appendix H. With-in Case Participant Verification, Pre-Training

Rivier UNIVERSITY

Participant Verification
Within-Case Directions

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193

Study Title: Teacher Beliefs about Quality Mathematics Lessons

Dear ______________________,

Thank you for your continued participation in this study regarding teacher beliefs about quality mathematics lessons. I have attached my write-up of your beliefs about quality mathematics lessons as I understand them from the data I have collected thus far. I would like your feedback so I can correct the inaccuracies in my work. All your responses are part of your participation in this research study and your confidentiality will be protected.

Please read the write-up of your beliefs with pen in hand. Mark any initial thoughts, good or bad, in the margins. (If you prefer to work electronically you may use the comments feature in Word.)

Please respond to this email with a time when we can meet to discuss your thoughts, reactions, and changes. If you prefer to meet in cyberspace we can discuss your thoughts via Skype. Contact me to set up a time. I have included the questions we will be discussing below so that you can think about them beforehand. I look forward to discussing them with you. Thank you!

Participant Verification Questions:

1. Are there places where I got it right, where you feel that what is written is particularly accurate?

2. Are there places where I got it wrong? Please help me correct my misunderstandings.

3. Are there things I left out? Is there anything I should add so that this write-up better reflects your beliefs?

Appendix I. CCSS Domains Included in UEM

Common Core State Standards (CCSS) for Mathematics K-8 Domains

<table>
<tr><td>K</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td colspan="9">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td></tr>
<tr><td colspan="6">Numbers and Operations in Base Ten</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td></tr>
<tr><td>Counting & Cardinality</td><td colspan="2"></td><td colspan="3">Numbers and Operations - Fractions</td><td colspan="2">Ratios and Proportional Relationships</td><td>Functions</td></tr>
</table>

= Domain covered in Understanding Elementary Mathematics (UEM) professional development program.

Graphic copied and used under creative commons license from: Illustrative Mathematics. (n.d.). *Content standards: kindergarten through grade 8*. Retrieved from http://www.illustrativemathematics.org/standards/k8

Appendix J. Content and Organization of UEM

The eight days of the Understanding Elementary Mathematics (UEM) professional development program contained 17 work sessions each 1 hour and 25 minutes in length. In addition, UEM contained two sessions for pre- and post-testing (days 1 and 8), two reflection sessions (days 5 and 8), and one 35-minute session devoted to a summary of the standards for mathematical practice (day 8). A brief description of the focus of each work session is in the table below.

Work Session (WS)	***Focal Area***
Day 1, WS 1	The Common Core State Standards: Content and Practice Standards
Day 1, WS 2	An Introduction to Number
Day 2, WS 1	Parts and Pieces
Day 2, WS 2	Addition of Whole Numbers and Decimals
Day 3, WS 1	Addition of Fractions
Day 3, WS 2	Addition of Signed Numbers
Day 4, WS 1	Subtraction of Whole Numbers and Decimals
Day 4, WS 2	Subtraction of Fractions
Day 5, WS 1	Subtraction of Signed Numbers
Day 5, WS 2	Multiplication of Whole Numbers
Day 5, WS 3	Factors and Prime Factorization
Day 6, WS 1	Multiplication of Fractions
Day 6, WS 2	Multiplication of Fractions (cont.)
Day 7, WS 1	Division of Whole Numbers
Day 7, WS 2	Multiplication and Division of Signed Numbers
Day 8, WS 1	Division of Fractions
Day 8, WS 2	Division of Fractions (cont.)

Appendix K. Training Phase Journal Directions

Rivier UNIVERSITY

Teacher Submissions
Training Phase Journal Directions

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193
Study Title: Teacher Beliefs about Quality Mathematics Lessons

Dear ________________________,

Thank you for your continued participation in this study regarding teacher beliefs about quality mathematics lessons. All submissions are part of your participation in this research study and your confidentiality will be protected.

Please keep a journal as you participate in Understanding Elementary Mathematics. The purpose of the journal is for you to reflect on your beliefs about quality mathematics lessons. *You may reflect in your journal as often as you like, but please be sure to respond to the focus questions below **at least once during each time period***. Please date each journal entry and label it with your pseudonym.

When to submit your entries: Please submit all journal entries to date at the end of each time period below.

Time Period One: from February 10th to February 20th

Time Period Two: from February 21st to March 13th

Time Period Three: from March 14th to March 24th

How to submit your entries: Send your journals to me either electronically (agaffney@rivier.edu), hand them to me in class, or by mailing a hard copy (Ann Gaffney, 28 Tokanel Dr., Londonderry, NH 03053). Thank you!

Focus Questions:

1. What makes a math lesson “high quality”?

2. Has what you are learning in Understanding Elementary Mathematics reinforced your beliefs about high quality mathematics lessons? changed them? In what ways? Give examples when possible.

3. If you had to define the phrase “quality mathematics lesson,” how would you define it?

4. What would be key characteristics of a quality mathematics lesson? What might one see, hear, or notice when observing a quality mathematics lesson?

Appendix L. With-in Case Participant Verification, Post-Training

Rivier UNIVERSITY

Participant Verification
Within-Case Directions 2

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193

Study Title: Teacher Beliefs about Quality Mathematics Lessons

Dear ______________________,

Thank you for your continued participation in this study regarding teacher beliefs about quality mathematics lessons. I have attached my write-up of your beliefs about quality mathematics lessons as I understand them from the data I collected during the study. I would like your feedback so I can correct the inaccuracies in my work. All your responses are part of your participation in this research study and your confidentiality will be protected.

Please read the write-up of your beliefs with pen in hand. Mark any initial thoughts, good or bad, in the margins. (If you prefer to work electronically you may use the comments feature in Word.)

Please respond to this email with your thoughts, reactions, and changes. Please make sure your responses include the answers to the questions listed below. Thank you!

Participant Verification Questions:

1. Are there places where I got it right, where you feel that what is written is particularly accurate?

2. Are there places where I got it wrong? Please help me correct my misunderstandings.

3. Are there things I left out? Is there anything I should add so that this write-up better reflects your beliefs?

Appendix M. Cross-Case Participant Verification Directions

Rivier UNIVERSITY

Participant Verification
Cross-Case Directions

Researcher: Ann Gaffney
agaffney@rivier.edu
603-490-7193

Study Title: Teacher Beliefs about Quality Mathematics Lessons

Dear ___[Use pseudonym here]__,

We have reached the last piece of your participation in this study regarding teacher beliefs about quality mathematics lessons. Thank you so much for your participation! You are about to see part of the results of the study.

Below is a link, username, and password to allow you access to a webpage with an online discussion forum. When you log in there will be directions about how to navigate within the system. ***Please log in sometime within the next week.*** The first time you log into the forum, plan to spend 15 minutes answering a quick (5 question) survey and just over 1 hour reading the results and responding to the discussion questions. During future logins you will read and respond to others' comments. These logins will probably take only about ten to fifteen minutes. ***Plan on logging in every few days for about two weeks***. I hope you enjoy the discussion!

Click here to go to the webpage.
Your username: [Insert username here]
Your password: [Insert password here]

Having trouble logging in? This webpage is a Google site. If your computer automatically logs you into Google (through Gmail or other Google applications) then you will need to log out and sign in using your study username and password. Only your study login will give you access to the site. This is to protect your confidentiality.

An important note about confidentiality:

This webpage and discussion forum is open only to participants in this study. Every participant has a pseudonym, so your identity and the identities of the other participants are still protected. By logging in to the webpage, you are agreeing not to print, save, email, or otherwise capture the comments of others. These comments are intended for use in this study only and must remain confidential. Thank you for your help protecting the confidentiality of all participants.

Appendix N. Cross-Case Participant Verification Questions

These questions were posted in the online discussion forum. Participants posted their own responses and commented on others' responses.

1. The piece you read pulls out the themes in responses from all participants in this study. Which themes do you find particularly reflective of your ways of thinking?
2. Are there themes in the write-up that are not reflective of you at all? Which ones?
3. Are there strong beliefs that you have about quality mathematics lessons that you did not see in the write-up? If so, post them here. Read others' responses. Do you agree with what they added? Why or why not?
4. Have your views changed when reading others' responses? In what ways?